$$y_{ij} = \mu + \tau_i + \beta(x_{ij} - \bar{x}_{..}) + \epsilon_{ij} \quad \begin{cases} i = 1, 2, 3 \\ j = 1, 2, \ldots, 5 \end{cases}$$

Using Equations 15-17 through 15-25, we may compute

$$S_{yy} = \sum_{i=1}^{3} \sum_{j=1}^{5} y_{ij}^2 - \frac{y_{..}^2}{an} = (36)^2 + (41)^2 + \cdots + (32)^2 - \frac{(603)^2}{(3)(5)} = 346.40$$

$$S_{xx} = \sum_{i=1}^{3} \sum_{j=1}^{5} x_{ij}^2 - \frac{x_{..}^2}{an} = (20)^2 + (25)^2 + \cdots + (15)^2 - \frac{(362)^2}{(3)(5)} = 261.73$$

$$S_{xy} = \sum_{i=1}^{3} \sum_{j=1}^{5} x_{ij} y_{ij} - \frac{(x_{..})(y_{..})}{an} = (20)(36) + (25)(41) + \cdots + (15)(32) - \frac{(362)(603)}{(3)(5)} = 282.60$$

$$T_{yy} = \frac{1}{n} \sum_{i=1}^{3} y_{i.}^2 - \frac{y_{..}^2}{an} = \frac{1}{5}[(207)^2 + (216)^2 + (180)^2] - \frac{(603)^2}{(3)(5)} = 140.40$$

$$T_{xx} = \frac{1}{n} \sum_{i=1}^{3} x_{i.}^2 - \frac{x_{..}^2}{an} = \frac{1}{5}[(126)^2 + (130)^2 + (106)^2] - \frac{(362)^2}{(3)(5)} = 66.13$$

$$T_{xy} = \frac{1}{n} \sum_{i=1}^{3} x_{i.} y_{i.} - \frac{(x_{..})(y_{..})}{an} = \frac{1}{5}[(126)(207) + (130)(216) + (106)(184)] - \frac{(362)(603)}{(3)(5)} = 96.00$$

$$E_{yy} = S_{yy} - T_{yy} = 346.40 - 140.40 = 206.00$$
$$E_{xx} = S_{xx} - T_{xx} = 261.73 - 66.13 = 195.60$$
$$E_{xy} = S_{xy} - T_{xy} = 282.60 - 96.00 = 186.60$$

From Equation 15-29, we find

$$\begin{aligned} SS_E' &= S_{yy} - (S_{xy})^2/S_{xx} \\ &= 346.40 - (282.60)^2/261.73 \\ &= 41.27 \end{aligned}$$

with $an - 2 = (3)(5) - 2 = 13$ degrees of freedom; and from Equation 15-27, we find

$$\begin{aligned} SS_E &= E_{yy} - (E_{xy})^2/E_{xx} \\ &= 206.00 - (186.60)^2/195.60 \\ &= 27.99 \end{aligned}$$

with $a(n - 1) - 1 = 3(5 - 1) - 1 = 11$ degrees of freedom.

The sum of squares for testing $H_0\colon \tau_1 = \tau_2 = \tau_3 = 0$ is

$$\begin{aligned} SS_E' - SS_E &= 41.27 - 27.99 \\ &= 13.28 \end{aligned}$$

with $a - 1 = 3 - 1 = 2$ degrees of freedom. These calculations are summarized in Table 15-13.