BAB 7:

RANCANGAN FAKTORIAL

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Sources:
Montgomery, Douglas C., Design and Analysis of Experiments, 6th Ed, John Wiley & Sons, New York, 2005

Perancangan Eksperimen

Bab 7:

Rancangan Faktorial

Bacaan:

- Montgomery bab 5
- www.teknikindustri.org

Topik

1. Definisi dan Prinsip Dasar
   - Efek utama dan efek interaksi
   - Percobaan faktorial dengan/tanpa interaksi
   - Permukaan respon dan plot kontur
2. Rancangan Faktorial Dua-Faktor
   - Hipotesis
   - ANOVA
   - Duncan
   - Uji Kecukupan Model
   - Ukuran Sampel
3. Rancangan Faktorial Umum

1. Definisi dan Prinsip Dasar

Efek faktor: perubahan dalam rata-rata respon dimana faktor diubah dari rendah ke tinggi

Efek utama *(main effect)* dari sebuah faktor: adalah kontras antar level dalam satu faktor, rata-rata terhadap level dari faktor lain

Efek interaksi *(interaction effect)* antar dua faktor: perbedaan antara *simple effect* dari satu faktor pada level yang berbeda dari faktor lain

Rancangan faktorial tanpa interaksi
(A factorial experiment without interaction)

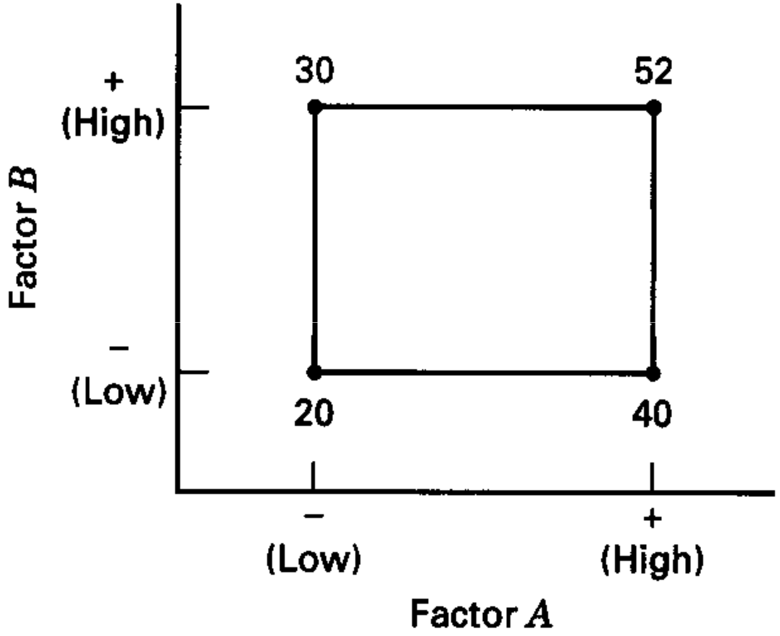

Figure 5-1 A two-factor factorial experiment, with the response (y) shown at the corners.

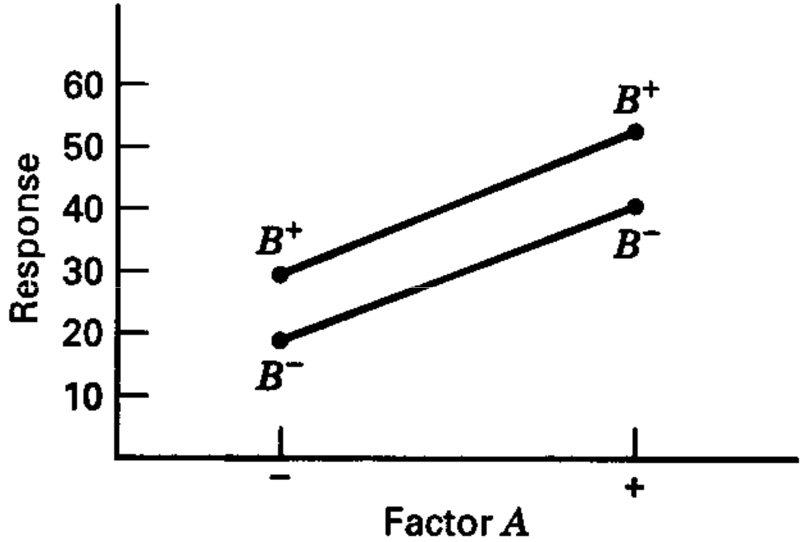

Figure 5-3 A factorial experiment without interaction.

Rancangan faktorial tanpa interaksi
(A factorial experiment without interaction)

Efek utama

$$A = \bar{y}_{A^+} - \bar{y}_{A^-} = \frac{40+52}{2} - \frac{20+30}{2} = 21$$

$$B = \bar{y}_{B^+} - \bar{y}_{B^-} = \frac{30+52}{2} - \frac{20+40}{2} = 11$$

Efek interaksi

$$AB = \frac{52+20}{2} - \frac{30+40}{2} = -1$$

Rancangan faktorial dengan interaksi
(A factorial experiment with interaction)

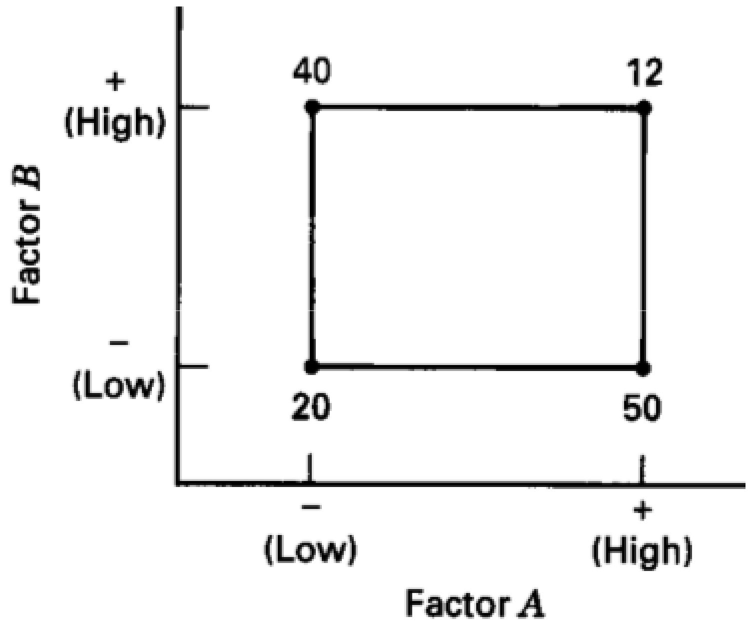

Figure 5-2 A two-factor factorial experiment with interaction.

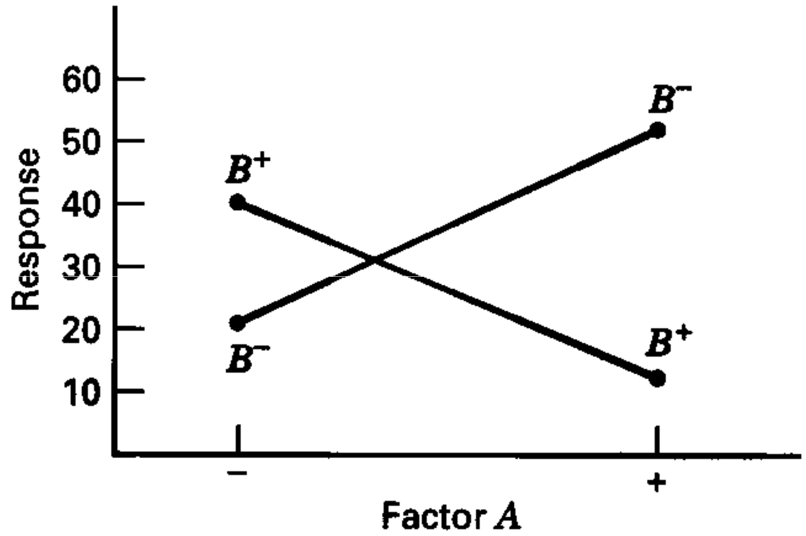

Figure 5-4 A factorial experiment with interaction.

Rancangan faktorial dengan interaksi
(A factorial experiment with interaction)

Efek utama

$$A = \bar{y}_{A^+} - \bar{y}_{A^-} = \frac{50+12}{2} - \frac{20+40}{2} = -1$$

$$B = \bar{y}_{B^+} - \bar{y}_{B^-} = \frac{40+12}{2} - \frac{20+50}{2} = 1$$

Efek interaksi

$$AB = \frac{12+20}{2} - \frac{40+50}{2} = -29$$

Permukaan respon dan plot kontur *(Response surface and contour plot)*

- Cara lain untuk menggambarkan konsep interaksi adalah melalui permukaan respon dan plot kontur
- Dapat diterapkan pada faktor-faktor rancangan kuantitatif
- Gunakan representasi model regresi untuk mem-plot permukaan respon dan kontur
- $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \varepsilon$

Permukaan respon dan plot kontur
(Response surface and contour plot)

Lihat gambar 5.1:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \varepsilon$$

β: parameter

Variable represent factor A

Variable represent factor B

Interaction between factors

Random error

Permukaan respon dan plot kontur
(Response surface and contour plot)

Ref: fig 5.1

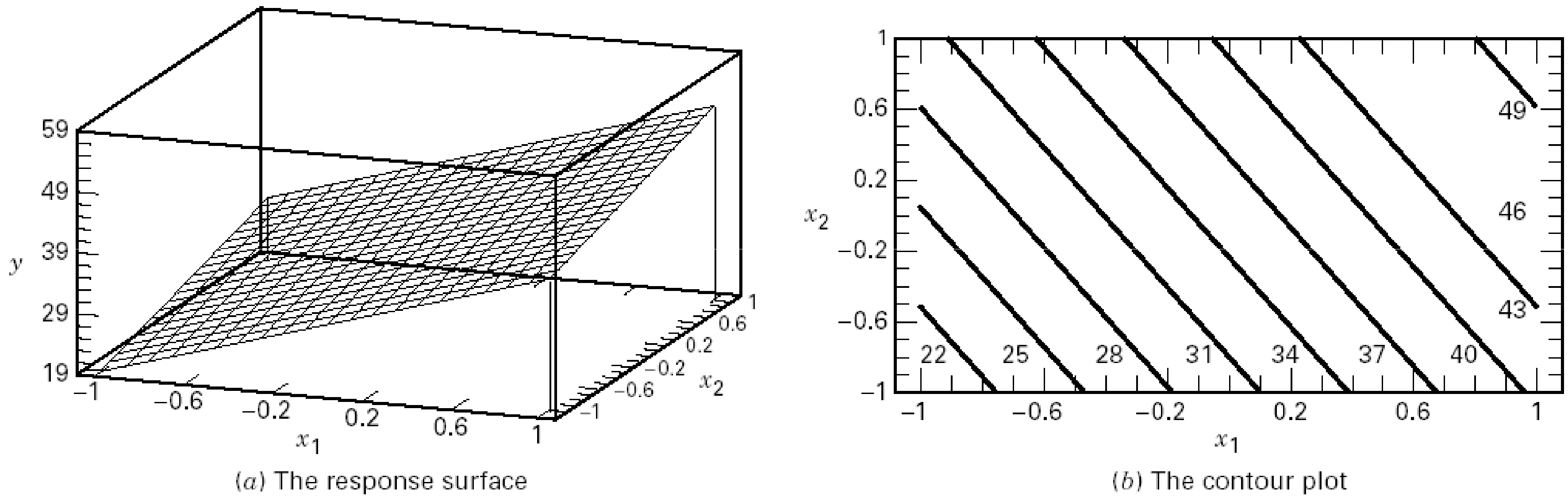

Figure 5-5 Response surface and contour plot for the model $\hat{y} = 35.5 + 10.5x_1 + 5.5x_2$.

$\beta_1 = 21/2 = 10.5$

$\beta_2 = 11/2 = 5.5$

$\beta_{12} = 1/2 = 0.5$ → abaikan

$\beta_0 = (20+40+30+52)/4 = 35.5$

Permukaan respon dan plot kontur
(Response surface and contour plot)

Ref: fig 5.1

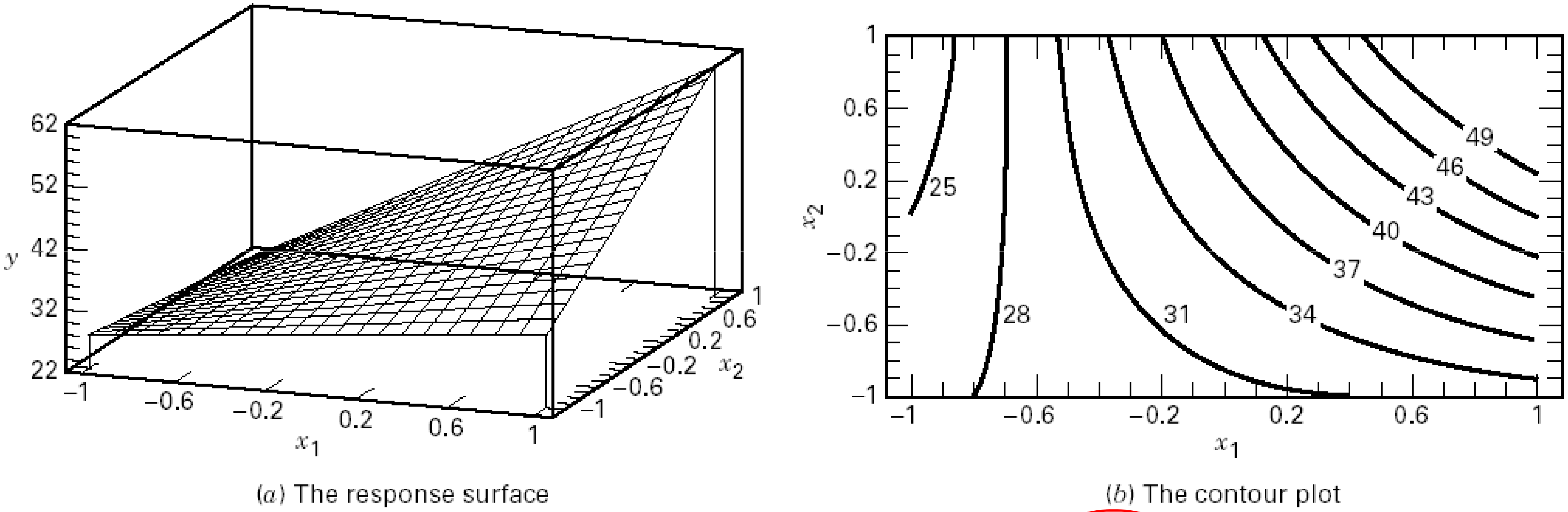

Figure 5-6 Response surface and contour plot for the model $\hat{y} = 35.5 + 10.5x_1 + 5.5x_2 + 8x_1x_2$.

Misalkan efek interaksi tidak diabaikan

bentuk lengkungan *(curvature)* menggambarkan interaksi

Keuntungan Faktorial

- Lebih efisien daripada eksperimen *one-factor-at-a-time*
- Menghindari konklusi yang salah bila ternyata terdapat interaksi antar faktor-faktornya
- Rancangan faktorial memungkinkan kita melakukan estimasi efek sebuah faktor pada beberapa level faktor yang lain

2. Rancangan Faktorial Dua-Faktor
(The two-factor factorial design)

2. Rancangan Faktorial 2 Faktor

Table 5-2 General Arrangement for a Two-Factor Factorial Design

		Factor B			
		1	2	...	b
Factor A	1	$y_{111}, y_{112}, \ldots, y_{11n}$	$y_{121}, y_{122}, \ldots, y_{12n}$		$y_{1b1}, y_{1b2}, \ldots, y_{1bn}$
	2	$y_{211}, y_{212}, \ldots, y_{21n}$	$y_{221}, y_{222}, \ldots, y_{22n}$		$y_{2b1}, y_{2b2}, \ldots, y_{2bn}$
	⋮				
	a	$y_{a11}, y_{a12}, \ldots, y_{a1n}$	$y_{a21}, y_{a22}, \ldots, y_{a2n}$		$y_{ab1}, y_{ab2}, \ldots, y_{abn}$

a level dari faktor *A*; *b* level faktor *B*; *n* replikasi

Ini disebut **rancangan acak lengkap *(completely randomized design)***

The two-factor factorial design

Model Statistik (Efek):

$$y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \varepsilon_{ijk} \begin{cases} i = 1,2,\ldots,a \\ j = 1,2,\ldots,b \\ k = 1,2,\ldots,n \end{cases}$$

- μ: Efek rata-rata keseluruhan
- τ_i: Efek pada level-*i* dari faktor baris A
- β_j: Efek pada level-*j* dari faktor kolom B
- $(\tau\beta)_{ij}$: Efek interaksi antara τ_i dan β_j
- ε_{ijk}: *Random error*

Uji Hipotesis

- Efek utama A:

$$H_0 : \tau_1 = \tau_2 = \ldots = \tau_a = 0$$
$$H_1 : \text{at least one } \tau_i \neq 0$$

- Efek utama B:

$$H_0 : \beta_1 = \beta_2 = \ldots = \beta_b = 0$$
$$H_1 : \text{at least one } \beta_j \neq 0$$

- Efek interaksi A danB:

$$H_0 : (\tau\beta)_{ij} = 0 \text{ for all } i, j$$
$$H_1 : \text{at least one } (\tau\beta)_{ij} \neq 0$$

$$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(y_{ijk}-\bar{y}_{...})^2 = bn\sum_{i=1}^{a}(\bar{y}_{i..}-\bar{y}_{...})^2 + an\sum_{j=1}^{b}(\bar{y}_{.j.}-\bar{y}_{...})^2$$

$$+ n\sum_{i=1}^{a}\sum_{j=1}^{b}(\bar{y}_{ij.}-\bar{y}_{i..}-\bar{y}_{.j.}+\bar{y}_{...})^2 + \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(y_{ijk}-\bar{y}_{ij.})^2$$

$$SS_T = SS_A + SS_B + SS_{AB} + SS_E$$

$$abn-1 = (a-1)+(b-1)+(a-1)(b-1)+ab(n-1)$$

Tabel ANOVA:
Two-Factorial, Fixed Effects Model

Table 5-3 The Analysis of Variance Table for the Two-Factor Factorial, Fixed Effects Model

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
A treatments	SS_A	$a-1$	$MS_A = \frac{SS_A}{a-1}$	$F_0 = \frac{MS_A}{MS_E}$
B treatments	SS_B	$b-1$	$MS_B = \frac{SS_B}{b-1}$	$F_0 = \frac{MS_B}{MS_E}$
Interaction	SS_{AB}	$(a-1)(b-1)$	$MS_{AB} = \frac{SS_{AB}}{(a-1)(b-1)}$	$F_0 = \frac{MS_{AB}}{MS_E}$
Error	SS_E	$ab(n-1)$	$MS_E = \frac{SS_E}{ab(n-1)}$	
Total	SS_T	$abn-1$		

Tabel ANOVA:
Two-Factorial, Fixed Effects Model

$$SS_T = \sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{n} y_{ijk}^2 - \frac{y_{...}^2}{abn}$$

$$SS_A = \frac{1}{bn} \sum_{i=1}^{a} y_{i..}^2 - \frac{y_{...}^2}{abn}$$

$$SS_B = \frac{1}{an} \sum_{j=1}^{b} y_{.j.}^2 - \frac{y_{...}^2}{abn}$$

$$SS_{\text{Subtotals}} = \frac{1}{n} \sum_{i=1}^{a} \sum_{j=1}^{b} y_{ij.}^2 - \frac{y_{...}^2}{abn}$$

Tabel ANOVA:
Two-Factorial, Fixed Effects Model

$$SS_{AB} = SS_{\text{Subtotals}} - SS_A - SS_B$$

$$SS_E = SS_T - SS_{AB} - SS_A - SS_B$$

$$SS_E = SS_T - SS_{\text{Subtotals}}$$

Contoh Kasus:

Life Data for the Battery Design

Table 5-1 Life (in hours) Data for the Battery Design Example

Material Type	Temperature (°F)					
	15		70		125	
1	130	155	34	40	20	70
	74	180	80	75	82	58
2	150	188	136	122	25	70
	159	126	106	115	58	45
3	138	110	174	120	96	104
	168	160	150	139	82	60

Contoh Kasus:
Life Data for the Battery Design

Response: Life in hours
ANOVA for Selected Factorial Model
Analysis of variance table [Partial sum of squares]

Source	**Sum of Squares**	**DF**	**Mean Square**	***F* Value**	**Prob > *F***	
Model	59416.22	8	7427.03	11.00	<0.0001	significant
A	*10683.72*	*2*	*5341.86*	*7.91*	*0.0020*	
B	*39118.72*	*2*	*19559.36*	*28.97*	*<0.0001*	
AB	*9613.78*	*4*	*2403.44*	*3.56*	*0.0186*	
Residual	18230.75	27	675.21			
Lack of Fit	*0.000*	*0*				
Pure Error	*18230.75*	*27*	*675.21*			
Cor Total	77646.97	35				

Std. Dev.	25.98	R-Squared	0.7652
Mean	105.53	Adj R-Squared	0.6956
C.V.	24.62	Pred R-Squared	0.5826
PRESS	32410.22	Adeq Precision	8.178

Uji Kecukupan Model
(Model Adequacy Checking)

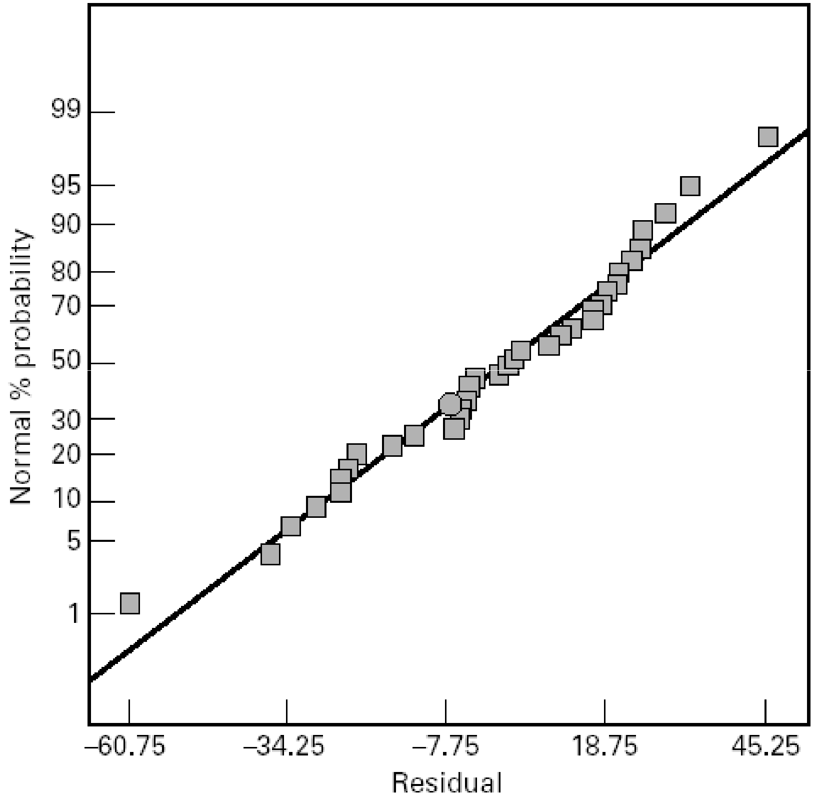

Figure 5-11 Normal probability plot of residuals for Example 5-1.

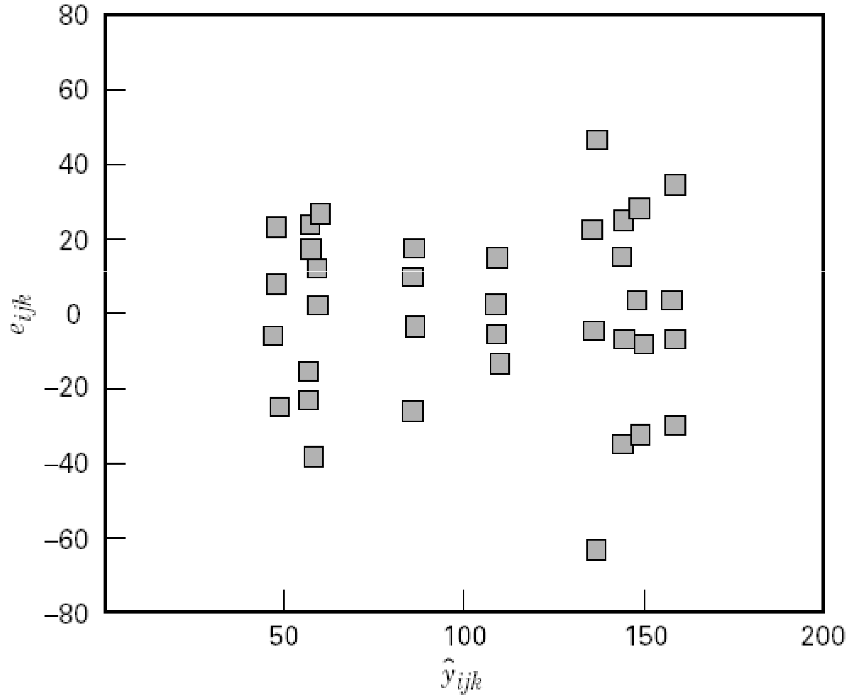

Figure 5-12 Plot of residuals versus $\hat{y}_{ijk}$ for Example 5-1.

Uji Kecukupan Model
(Model Adequacy Checking)

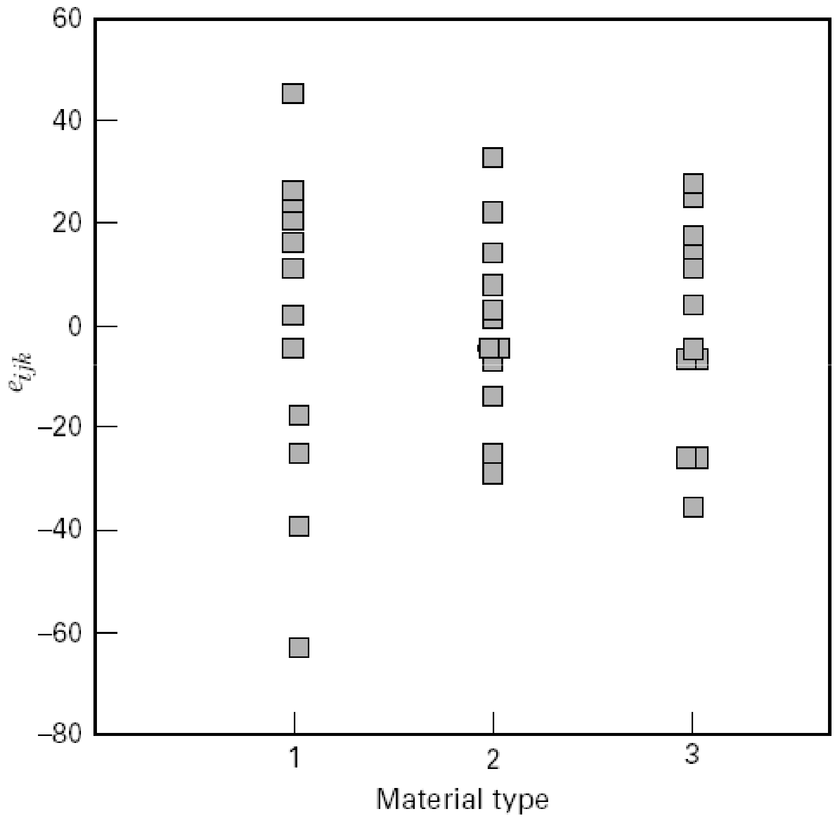

Figure 5-13 Plot of residuals versus material type for Example 5-1.

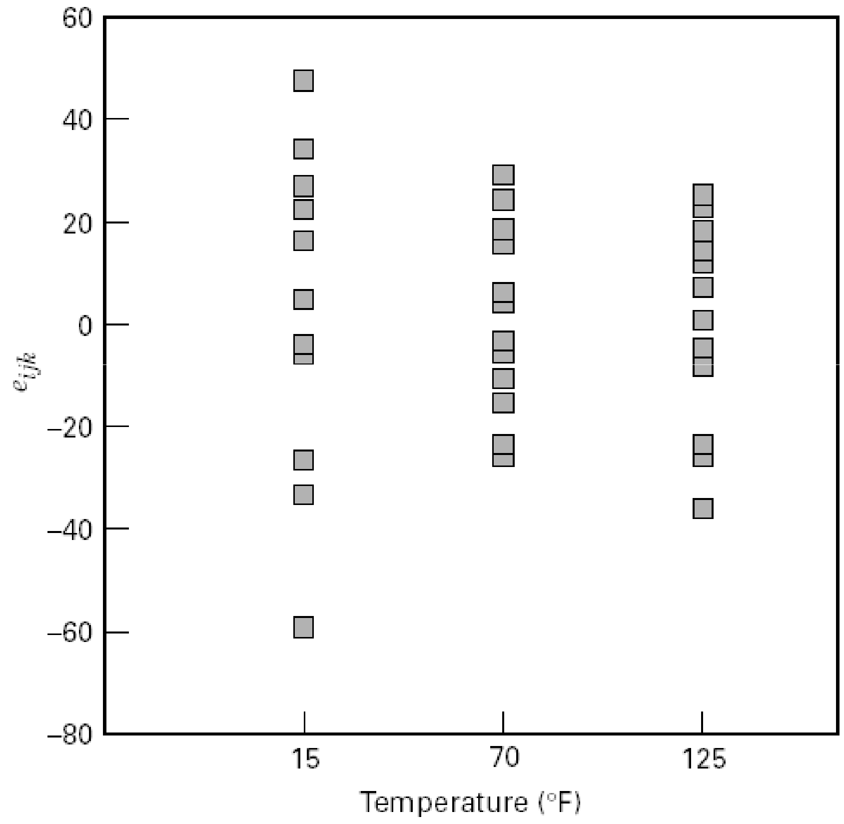

Figure 5-14 Plot of residuals versus temperature for Example 5-1.

3. Rancangan Faktorial Umum
(The General Factorial Design)

3. Rancangan Faktorial Umum
(The General Factorial Design)

- Prosedur dasar sama dengan kasus dua-faktor; seluruh *abc...kn* kombinasi percobaan dilaksanakan dalam urutan acak
- Pembagian *sum of square* juga sama:

$$SS_T = SS_A + SS_B + \ldots + SS_{AB} + SS_{AC} + \ldots + SS_{ABC} + \ldots + SS_{AB\cdots K} + SS_E$$

The ANOVA Table for the Three-Factor Fixed Effects Model

Table 5-12 The Analysis of Variance Table for the Three-Factor Fixed Effects Model

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	Expected Mean Square	F_0
A	SS_A	$a-1$	MS_A	$\sigma^2 + \frac{bcn\sum \tau_i^2}{a-1}$	$F_0 = \frac{MS_A}{MS_E}$
B	SS_B	$b-1$	MS_B	$\sigma^2 + \frac{acn\sum \beta_j^2}{b-1}$	$F_0 = \frac{MS_B}{MS_E}$
C	SS_C	$c-1$	MS_C	$\sigma^2 + \frac{abn\sum \gamma_k^2}{c-1}$	$F_0 = \frac{MS_C}{MS_E}$
AB	SS_{AB}	$(a-1)(b-1)$	MS_{AB}	$\sigma^2 + \frac{cn\sum\sum (\tau\beta)_{ij}^2}{(a-1)(b-1)}$	$F_0 = \frac{MS_{AB}}{MS_E}$
AC	SS_{AC}	$(a-1)(c-1)$	MS_{AC}	$\sigma^2 + \frac{bn\sum\sum (\tau\gamma)_{ik}^2}{(a-1)(c-1)}$	$F_0 = \frac{MS_{AC}}{MS_E}$
BC	SS_{BC}	$(b-1)(c-1)$	MS_{BC}	$\sigma^2 + \frac{an\sum\sum (\beta\gamma)_{jk}^2}{(b-1)(c-1)}$	$F_0 = \frac{MS_{BC}}{MS_E}$
ABC	SS_{ABC}	$(a-1)(b-1)(c-1)$	MS_{ABC}	$\sigma^2 + \frac{n\sum\sum\sum (\tau\beta\gamma)_{ijk}^2}{(a-1)(b-1)(c-1)}$	$F_0 = \frac{MS_{ABC}}{MS_E}$
Error	SS_E	$abc(n-1)$	MS_E	σ^2	
Total	SS_T	$abcn-1$			

The ANOVA Table for the Three-Factor Fixed Effects Model (cont)

$$SS_{AB} = \frac{1}{cn}\sum_{i=1}^{a}\sum_{j=1}^{b} y_{ij..}^2 - \frac{y_{....}^2}{abcn} - SS_A - SS_B$$
$$= SS_{\text{Subtotals}(AB)} - SS_A - SS_B$$

$$SS_{AC} = \frac{1}{bn}\sum_{i=1}^{a}\sum_{k=1}^{c} y_{i.k.}^2 - \frac{y_{....}^2}{abcn} - SS_A - SS_C$$
$$= SS_{\text{Subtotals}(AC)} - SS_A - SS_C$$

$$SS_{BC} = \frac{1}{an}\sum_{j=1}^{b}\sum_{k=1}^{c} y_{.jk.}^2 - \frac{y_{....}^2}{abcn} - SS_B - SS_C$$
$$= SS_{\text{Subtotals}(BC)} - SS_B - SS_C$$

The ANOVA Table for the Three-Factor Fixed Effects Model (cont)

$$SS_{ABC} = \frac{1}{n}\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} y_{ijk.}^2 - \frac{y_{....}^2}{abcn} - SS_A - SS_B - SS_C - SS_{AB} - SS_{AC} - SS_{BC}$$
$$= SS_{\text{Subtotals}(ABC)} - SS_A - SS_B - SS_C - SS_{AB} - SS_{AC} - SS_{BC}$$

$$SS_E = SS_T - SS_{\text{Subtotals}(ABC)}$$

Contoh kasus:
Soft Drink Bottling Problem

Pembuat minuman ringan tertarik mengamati kesamaan tinggi cairan dalam botol yang diproduksi oleh pabriknya.

Terdapat tiga faktor:

A. Persen karbonasi: 10%, 12% and 14%
B. Tekanan dalam pengisi: 25 and 30 psi
C. Botol yang diproduksi per menit atau kecepatan produksi botol: 200 and 250 bpm

Ia melakukan dua replikasi sebuah rancangan faktorial

Contoh kasus: *Soft Drink Bottling Problem*

Percent carbonation (A)	Operating Pressure (B)				
	25 psi		30 psi		
	Line Speed (C)		Line Speed (C)		
	200	300	200	300	
10	-3	-1	-1	1	-4
	-1	0	0	1	
12	0	2	2	6	20
	1	1	3	5	
14	5	7	7	10	59
	4	6	9	11	
B X C totals	6	15	20	34	75
	21		54		