Battery Design Experiment

Table 5-4 Life Data (in hours) for the Battery Design Experiment

Material Type	Temperature (°F) 15			70			125			$y_{i..}$
1	130 74	155 180	(539)	34 80	40 75	(229)	20 82	70 58	(230)	998
2	150 159	188 126	(623)	136 106	122 115	(479)	25 58	70 45	(198)	1300
3	138 168	110 160	(576)	174 150	120 139	(583)	96 82	104 60	(342)	1501
$y_{.j.}$		1738			1291			770		$3799 = y_{...}$

Using Equations 5-6 through 5-10, the sums of squares are computed as follows:

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} y_{ijk}^2 - \frac{y_{...}^2}{abn}$$

$$= (130)^2 + (155)^2 + (74)^2 + \cdots + (60)^2 - \frac{(3799)^2}{36} = 77{,}646.97$$

$$SS_{\text{Material}} = \frac{1}{bn}\sum_{i=1}^{a} y_{i..}^2 - \frac{y_{...}^2}{abn}$$

$$= \frac{1}{(3)(4)}[(998)^2 + (1300)^2 + (1501)^2] - \frac{(3799)^2}{36} = 10{,}683.72$$

$$SS_{\text{Temperature}} = \frac{1}{an}\sum_{j=1}^{b} y_{.j.}^2 - \frac{y_{...}^2}{abn}$$

$$= \frac{1}{(3)(4)}[(1738)^2 + (1291)^2 + (770)^2] - \frac{(3799)^2}{36} = 39{,}118.72$$

$$SS_{\text{Interaction}} = \frac{1}{n}\sum_{i=1}^{a}\sum_{j=1}^{b} y_{ij.}^2 - \frac{y_{...}^2}{abn} - SS_{\text{Material}} - SS_{\text{Temperature}}$$

$$= \frac{1}{4}[(539)^2 + (229)^2 + \cdots + (342)^2] - \frac{(3799)^2}{36} - 10{,}683.72$$

$$- 39{,}118.72 = 9613.78$$

and

$$SS_E = SS_T - SS_{\text{Material}} - SS_{\text{Temperature}} - SS_{\text{Interaction}}$$

$$= 77{,}646.97 - 10{,}683.72 - 39{,}118.72 - 9613.78 = 18{,}230.75$$

The ANOVA is shown in Table 5-5. Because $F_{0.05,4,27} = 2.73$, we conclude that there is a significant interaction between material types and temperature. Furthermore, $F_{0.05,2,27} = 3.35$.

Table 5-5 Analysis of Variance for Battery Life Data

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	P-Value
Material types	10,683.72	2	5,341.86	7.91	0.0020
Temperature	39,118.72	2	19,559.36	28.97	0.0001
Interaction	9,613.78	4	2,403.44	3.56	0.0186
Error	18,230.75	27	675.21		
Total	77,646.97	35			

Tabel Residual untuk contoh 5-1

Table 5-6 Residuals for Example 5-1

Material Type	Temperature (°F)					
	15		70		125	
1	−4.75	20.25	−23.25	−17.25	−37.50	12.50
	−60.75	45.25	22.75	17.75	24.50	0.50
2	−5.75	32.25	16.25	2.25	−24.50	20.50
	3.25	−29.75	−13.75	−4.75	8.50	−4.50
3	−6.00	−34.00	28.25	−25.75	10.50	18.50
	24.00	16.00	4.25	−6.75	−3.50	−25.50

Pemilihan Ukuran Sampel

Misalkan untuk contoh 5.1 (Battery Life Design), sebelum melakukan eksperimen kita memutuskan bahwa hipotesis nol ditolak bila probabilitas umur baterai diantara dua temperatur sebesar 40 jam. Jadi perbedaan sebesar $D = 40$ adalah signifikan secara teknis, dan bila diasumsikan bahwa standar deviasi umur baterai mendekati 25, maka persamaannya adalah sbb:

$$\Phi^2 = \frac{naD^2}{2b\sigma^2} = \frac{n(3)(40)^2}{2(3)(25)^2} = 1.28n$$

n	Φ^2	Φ	ν_1 = Numerator Degrees of Freedom	ν_2 = Error Degrees of Freedom	β
2	2.56	1.60	2	9	0.45
3	3.84	1.96	2	18	0.18
4	5.12	2.26	2	27	0.06

Untuk α=0.05, pilih n=4

Soft Drink Bottling Problem

The total corrected sum of squares is found from Equation 5-27 as

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{n} y_{ijkl}^2 - \frac{y_{....}^2}{abcn}$$

$$= 571 - \frac{(75)^2}{24} = 336.625$$

Table 5-13 Fill Height Deviation Data for Example 5-3

Percent Carbonation (A)	Operating Pressure (B): 25 psi, Line Speed (C): 200		Operating Pressure (B): 25 psi, Line Speed (C): 250		Operating Pressure (B): 30 psi, Line Speed (C): 200		Operating Pressure (B): 30 psi, Line Speed (C): 250		$y_{i...}$
10	−3 −1	(−4)	−1 0	(−1)	−1 0	(−1)	1 1	(2)	−4
12	0 1	(1)	2 1	(3)	2 3	(5)	6 5	(11)	20
14	5 4	(9)	7 6	(13)	7 9	(16)	10 11	(21)	59
$B \times C$ Totals $y_{.jk.}$	6		15		20		34		$75 = y_{....}$
$y_{.j..}$	21				54				

$A \times B$ Totals $y_{ij..}$

A	B: 25	B: 30
10	−5	1
12	4	16
14	22	37

$A \times C$ Totals $y_{i.k.}$

A	C: 200	C: 250
10	−5	1
12	6	14
14	25	34

$$SS_{\text{Carbonation}} = \frac{1}{bcn}\sum_{i=1}^{a} y_{i...}^2 - \frac{y_{....}^2}{abcn}$$

$$= \frac{1}{8}[(-4)^2 + (20)^2 + (59)^2] - \frac{(75)^2}{24} = 252.750$$

$$SS_{\text{Pressure}} = \frac{1}{acn}\sum_{j=1}^{b} y_{.j..}^2 - \frac{y_{....}^2}{abcn}$$

$$= \frac{1}{12}[(21)^2 + (54)^2] - \frac{(75)^2}{24} = 45.375$$

$$SS_{\text{Speed}} = \frac{1}{abn}\sum_{k=1}^{c} y_{..k.}^2 - \frac{y_{....}^2}{abcn}$$

$$= \frac{1}{12}[(26)^2 + (49)^2] - \frac{(75)^2}{24} = 22.042$$

$$SS_{AC} = \frac{1}{bn}\sum_{i=1}^{a}\sum_{k=1}^{c} y_{i.k.}^2 - \frac{y_{....}^2}{abcn} - SS_A - SS_C$$
$$= \frac{1}{4}[(-5)^2 + (1)^2 + (6)^2 + (14)^2 + (25)^2 + (34)^2] - \frac{(75)^2}{24}$$
$$- 252.750 - 22.042$$
$$= 0.583$$

$$SS_{BC} = \frac{1}{an}\sum_{j=1}^{b}\sum_{k=1}^{c} y_{.jk.}^2 - \frac{y_{....}^2}{abcn} - SS_B - SS_C$$
$$= \frac{1}{6}[(6)^2 + (15)^2 + (20)^2 + (34)^2] - \frac{(75)^2}{24} - 45.375 - 22.042$$
$$= 1.042$$

$$SS_{ABC} = \frac{1}{n}\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} y_{ijk.}^2 - \frac{y_{....}^2}{abcn} - SS_A - SS_B - SS_C - SS_{AB} - SS_{AC} - SS_{BC}$$
$$= \frac{1}{2}[(-4)^2 + (-1)^2 + (-1)^2 + \cdots + (16)^2 + (21)^2] - \frac{(75)^2}{24}$$
$$- 252.750 - 45.375 - 22.042 - 5.250 - 0.583 - 1.042$$
$$= 1.083$$

Finally, noting that

$$SS_{\text{Subtotals}(ABC)} = \frac{1}{n}\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} y_{ijk.}^2 - \frac{y_{....}^2}{abcn} = 328.125$$

we have

$$SS_E = SS_T - SS_{\text{Subtotals}(ABC)}$$
$$= 336.625 - 328.125$$
$$= 8.500$$

Table 5-14 Analysis of Variance for Example 5-3

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	P-Value
Percentage of carbonation (A)	252.750	2	126.375	178.412	<0.0001
Operating pressure (B)	45.375	1	45.375	64.059	<0.0001
Line speed (C)	22.042	1	22.042	31.118	0.0001
AB	5.250	2	2.625	3.706	0.0558
AC	0.583	2	0.292	0.412	0.6713
BC	1.042	1	1.042	1.471	0.2485
ABC	1.083	2	0.542	0.765	0.4867
Error	8.500	12	0.708		
Total	336.625	23			