Bab 8:

Rancangan Faktorial 2^k

Perancangan Eksperimen

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Sources:
Montgomery, Douglas C., Design and Analysis of Experiments, 6th Ed, John Wiley & Sons, New York, 2005

Bab 8:

Rancangan Faktorial 2^k

Bacaan:

- Montgomery bab 6
- www.teknikindustri.org

Topik

1. Rancangan 2^2
2. Rancangan 2^3
3. Rancangan 2^k
   - Replikasi tunggal
   - Penambahan titik pusat *(center points)* pada rancangan 2^k

Pendahuluan

- **Kasus khusus** rancangan faktorial umum: *k* factor, semuanya dalam dua **level**
- Dua level yang ada dalam faktor biasanya disebut sebagai level **rendah** dan **tinggi** (kuantitatif atau kualitatif)
- Digunakan dalam eksperimen pemilihan faktor / *factor screening experiments)*
- Membentuk satu *"building block"* dasar untuk rancangan eksperimental yang lain
- Merupakan Metoda khusus *(short-cut)* untuk analisis

1. Rancangan 2^2

Contoh: Proses Kimia

Sebuah penelitian **<u>efek</u>** konsentrasi reaktan dan jumlah katalis **<u>pada</u>** konversi *(yield)*

Faktor A: konsentrasi reaktan [15%, 25%]
Faktor B: katalis [1 lbs, 2 lbs.]

Contoh Kasus

Factor		Treatment	Replicate			
A	*B*	Combination	I	II	III	Total
−	−	*A* low, *B* low	28	25	27	80
+	−	*A* high, *B* low	36	32	32	100
−	+	*A* low, *B* high	18	19	23	60
+	+	*A* high, *B* high	31	30	29	90

A = konsentrasi reaktan

B = jumlah katalis

y = konversi

Contoh Kasus

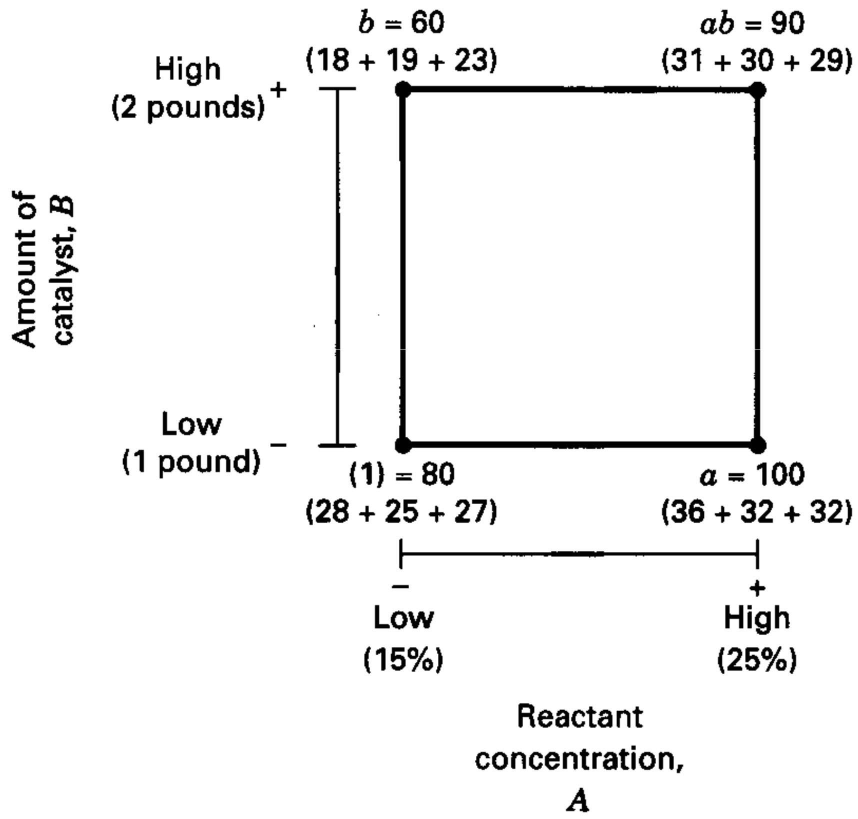


Figure 6-1 Treatment combinations in the 2^2 design.

$$A = \bar{y}_{A^+} - \bar{y}_{A^-}$$
$$= \frac{ab + a}{2n} - \frac{b + (1)}{2n}$$
$$= \frac{1}{2n}[ab + a - b - (1)]$$

$$B = \bar{y}_{B^+} - \bar{y}_{B^-}$$
$$= \frac{ab + b}{2n} - \frac{a + (1)}{2n}$$
$$= \frac{1}{2n}[ab + b - a - (1)]$$

$$AB = \frac{ab + (1)}{2n} - \frac{a + b}{2n}$$
$$= \frac{1}{2n}[ab + (1) - a - b]$$

Kontras

$$Contrast_A = ab + a - b - (1)$$

$$Contrast_B = ab + b - a - (1)$$

$$Contrast_{AB} = ab + (1) - a - b$$

Sum of Squares

$$SS_X = \frac{(Contrast_X)^2}{4n}$$

$$SS_A = \frac{[ab + a - b - (1)]^2}{4n} = \frac{50^2}{4(3)} = 208.33$$

$$SS_B = \frac{[ab + b - a - (1)]^2}{4n} = \frac{(-30)^2}{4(3)} = 75.00$$

$$SS_{AB} = \frac{[ab + (1) - a - b]^2}{4n} = \frac{10^2}{4(3)} = 8.33$$

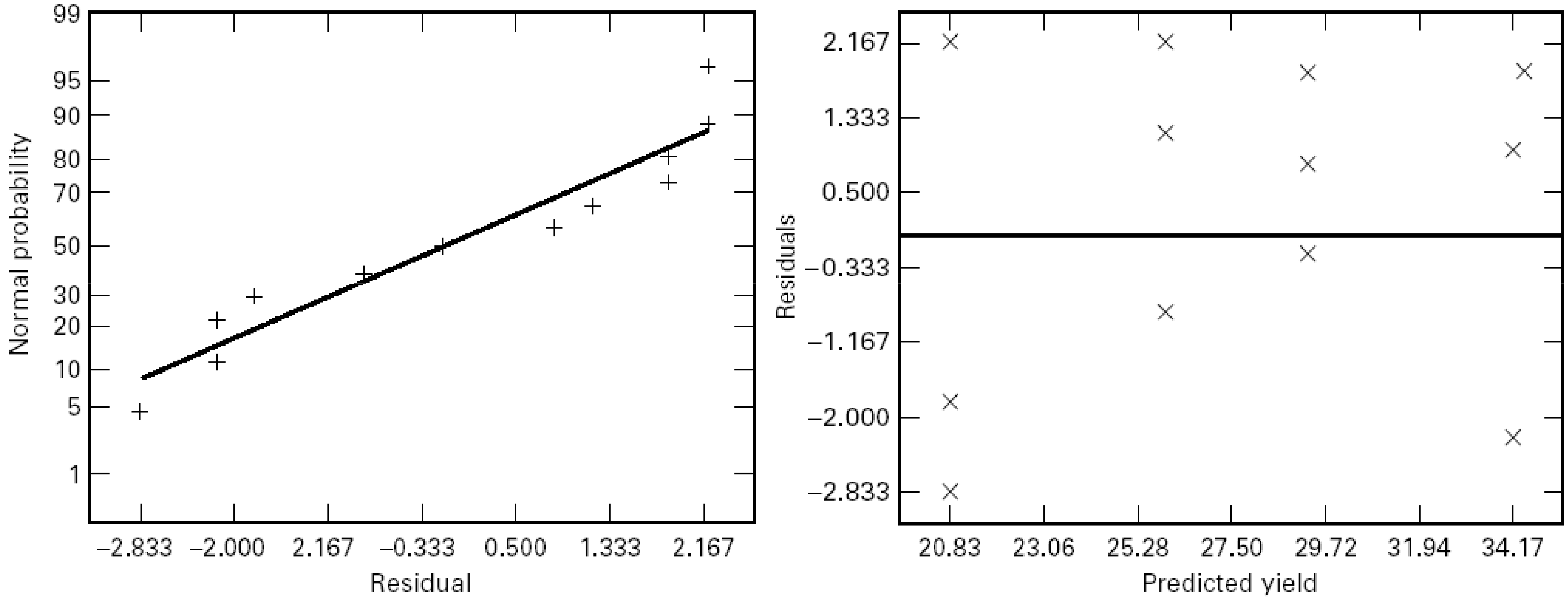
ANOVA

Table 6-1 Analysis of Variance for the Experiment in Figure 6-1

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	*P*-Value
A	208.33	1	208.33	53.15	0.0001
B	75.00	1	75.00	19.13	0.0024
AB	8.33	1	8.33	2.13	0.1826
Error	31.34	8	3.92		
Total	323.00	11			

$F_{.05;1,8} = 5.32$ and $F_{.01;1,8} = 11.26$

Adequacy Checking



(*a*) Normal probability plot

(*b*) Residuals versus predicted yield

Figure 6-2 Residual plots for the chemical process experiment.

Response surface and contour plot

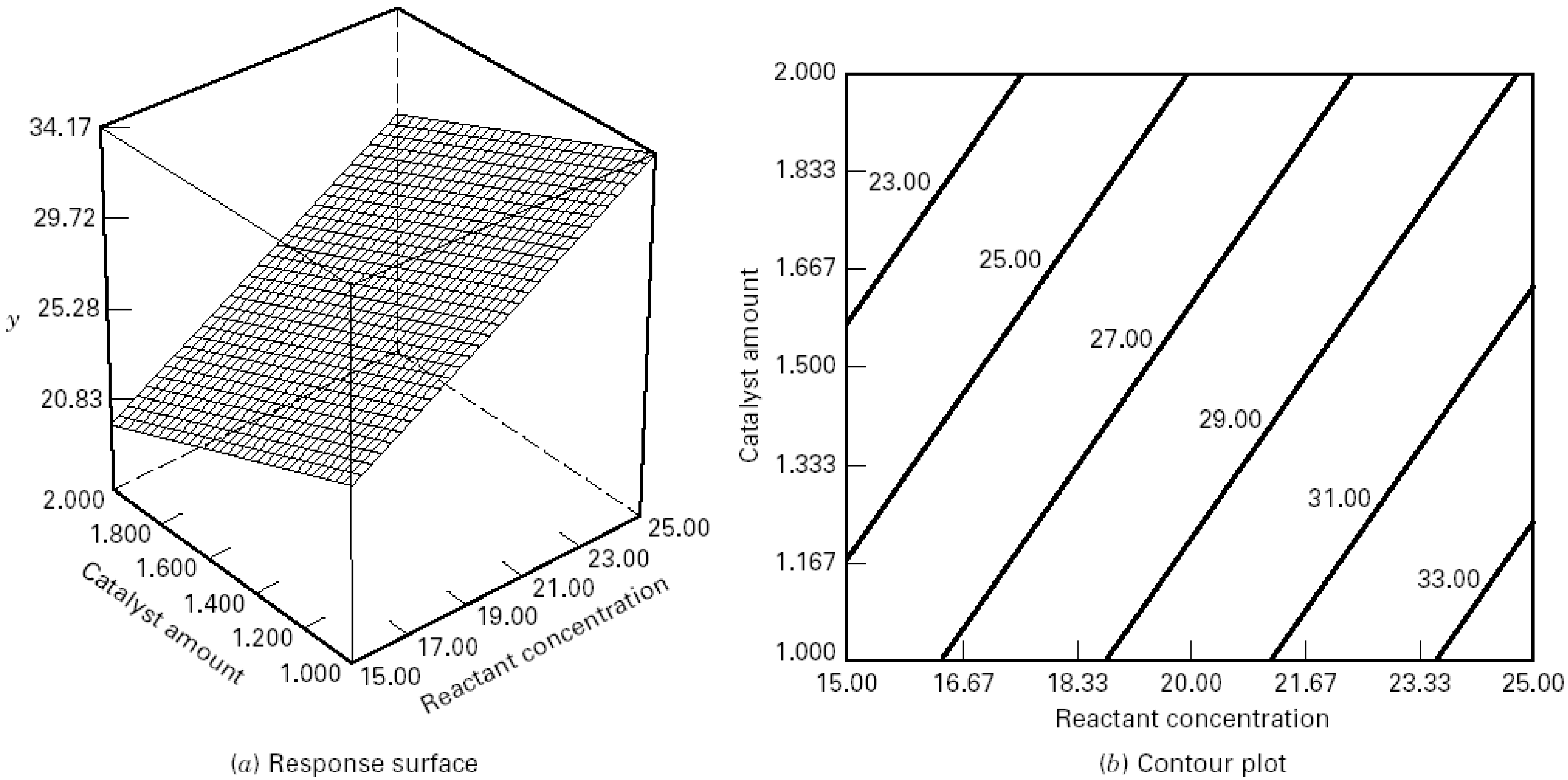
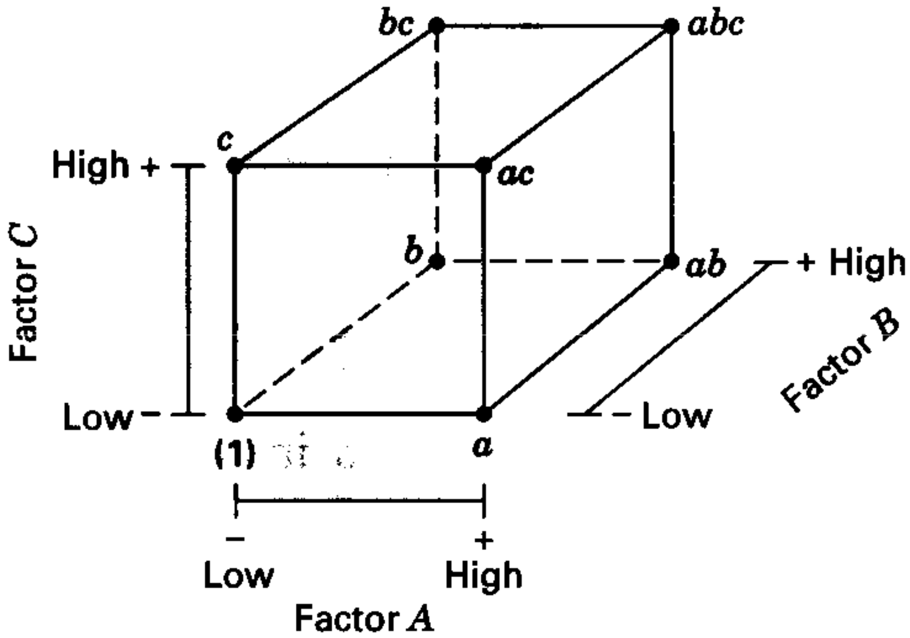


Figure 6-3 Response surface plot and contour plot of yield from the chemical process experiment.

2. Rancangan 2^3



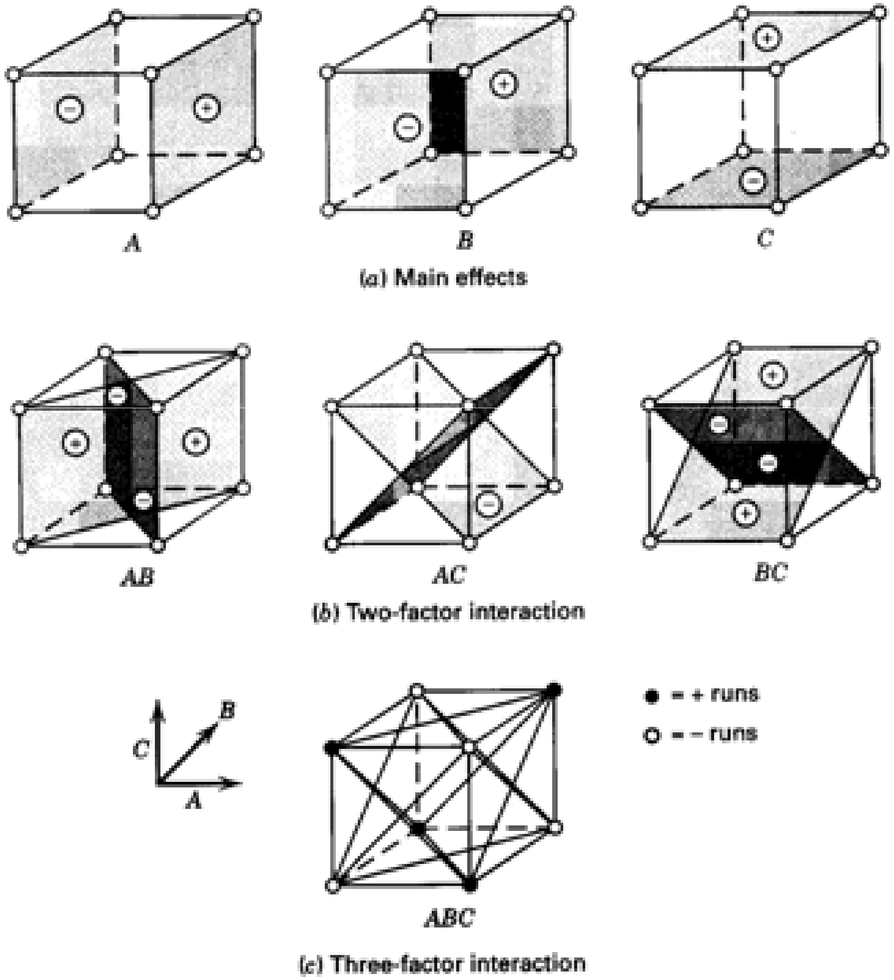
(a) Geometric view

Run	Factor A	Factor B	Factor C
1	−	−	−
2	+	−	−
3	−	+	−
4	+	+	−
5	−	−	+
6	+	−	+
7	−	+	+
8	+	+	+

(b) The design matrix

Figure 6-4 The 2^3 factorial design.

Rancangan 2^3



$$A = \bar{y}_{A^+} - \bar{y}_{A^-}$$

$$B = \bar{y}_{B^+} - \bar{y}_{B^-}$$

$$C = \bar{y}_{C^+} - \bar{y}_{C^-}$$

etc, etc, ...

Contoh

Table 6-4 The Plasma Etch Experiment, Example 6-1

	Coded Factors			Etch Rate			Factor Levels		
Run	A	B	C	Replicate 1	Replicate 2	Total		Low (−1)	High (+1)
1	−1	−1	−1	550	604	(1) = 1154	A (Gap, cm)	0.80	1.20
2	1	−1	−1	669	650	a = 1319	B (C_2F_6 flow, SCCM)	125	200
3	−1	1	−1	633	601	b = 1234	C (Power, W)	275	325
4	1	1	−1	642	635	ab = 1277			
5	−1	−1	1	1037	1052	c = 2089			
6	1	−1	1	749	868	ac = 1617			
7	−1	1	1	1075	1063	bc = 2178			
8	1	1	1	729	860	abc = 1589			

A = gap, B = Flow, C = Power, y = Etch Rate

Contoh (lanjutan)

Table 6-3 Algebraic Signs for Calculating Effects in the 2^3 Design

Treatment Combination	Factorial Effect							
	I	*A*	*B*	*AB*	*C*	*AC*	*BC*	*ABC*
(1)	+	−	−	+	−	+	+	−
a	+	+	−	−	−	−	+	+
b	+	−	+	−	−	+	−	+
ab	+	+	+	+	−	−	−	−
c	+	−	−	+	+	−	−	+
ac	+	+	−	−	+	+	−	−
bc	+	−	+	−	+	−	+	−
abc	+	+	+	+	+	+	+	+

Properti tabel

- Kecuali kolom *I*, setiap kolom memiliki angka sama bertanda + dan –
- Jumlah tanda dari dua kolom sama dengan nol
- Mengalikan kolom dengan ***I*** tidak akan mengubah kolom tersebut *(identity element)*
- Produk dari dua kolom menghasilkan satu kolom dalam tabel:

$$A \times B = AB$$

$$AB \times BC = AB^2C = AC$$

Kontras untuk Rancangan Faktorial 2^3

$$Contrast_A = a + ab + ac + abc - b - c - bc - (1)$$

$$Contrast_B = b + ab + bc + abc - a - c - ac - (1)$$

$$Contrast_C = c + ac + bc + abc - a - b - ab - (1)$$

$$Contrast_{AB} = c + ab + abc + (1) - a - b - ac - bc$$

$$Contrast_{AC} = b + ac + abc + (1) - a - c - ab - bc$$

$$Contrast_{BC} = a + bc + abc + (1) - b - c - ab - ac$$

$$Contrast_{ABC} = a + b + c + abc - ab - ac - bc - (1)$$

Efek dan *Sums of squares* untuk rancangan faktorial 2^3

$$\text{Estimasi Efek}_X = \frac{1}{4n}[\text{contrast}_X]$$

$$SS_X = \frac{(Contrast_X)^2}{8n}$$

Ringkasan Estimasi Efek

Table 6-5 Effect Estimate Summary for Example 6-1

Factor	Effect Estimate	Sum of Squares	Percent Contribution
A	−101.625	41,310.5625	7.7736
B	7.375	217.5625	0.0409
C	306.125	374,850.0625	70.5373
AB	−24.875	2475.0625	0.4657
AC	−153.625	94,402.5625	17.7642
BC	−2.125	18.0625	0.0034
ABC	5.625	126.5625	0.0238

Tabel ANOVA

Table 6-6 Analysis of Variance for the Plasma Etching Experiment

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	P-Value
Gap (A)	41,310.5625	1	41,310.5625	18.34	0.0027
Gas flow (B)	217.5625	1	217.5625	0.10	0.7639
Power (C)	374,850.0625	1	374,850.0625	166.41	0.0001
AB	2475.0625	1	2475.0625	1.10	0.3252
AC	94,402.5625	1	94,402.5625	41.91	0.0002
BC	18.0625	1	18.0625	0.01	0.9308
ABC	126.5625	1	126.5625	0.06	0.8186
Error	18,020.5000	8	2252.5625		
Total	531,420.9375	15			

3.a. Replikasi Tunggal dalam Rancangan 2^k

- Merupakan rancangan faktorial 2^k dengan satu pengamatan pada masing-masing sudut "kubus"
- Replikasi tunggal rancangan faktorial 2^k juga sering disebut 2^k yang "tidak direplikasi" *("unreplicated" of the* 2^k)
- Beresiko… jika hanya ada satu replikasi tiap sudut, apakah ada kemungkinan respon pengamatan yang tidak wajar merusak hasil percobaan?
- *Modeling "noise"?*

Spacing of Factor Levels in the Unreplicated 2^k Factorial Designs

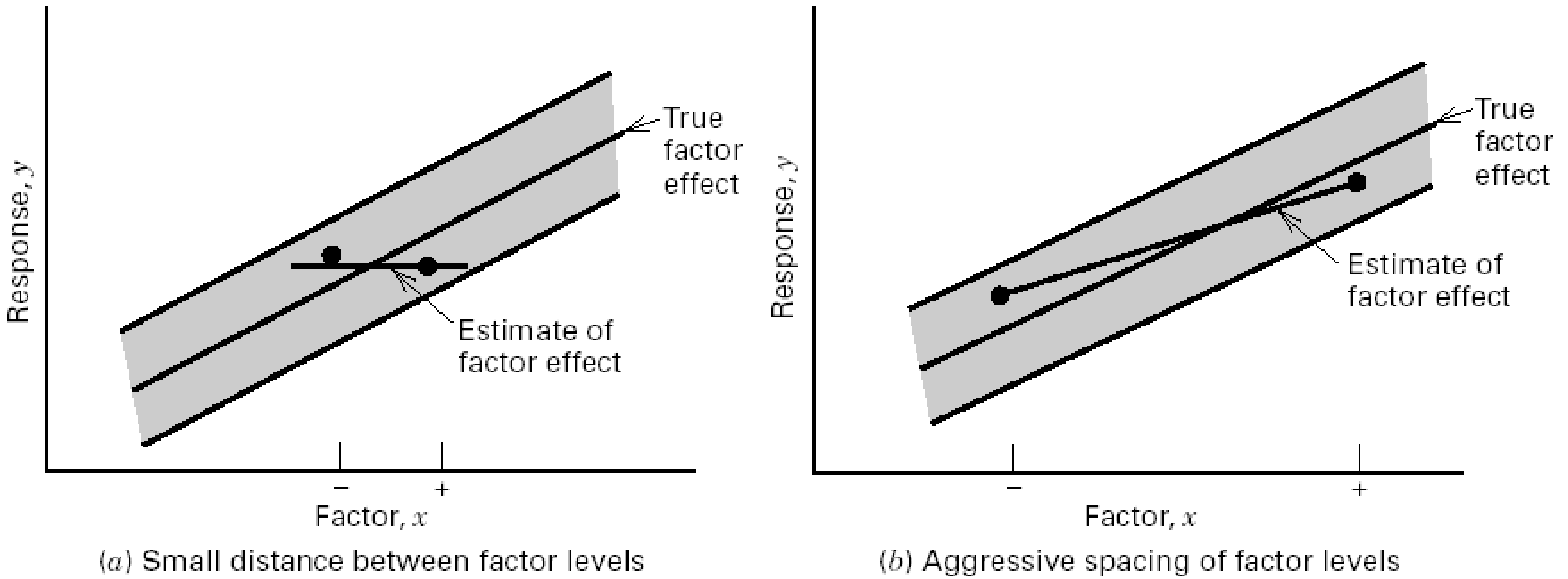


Figure 6-9 The impact of the choice of factor levels in an unreplicated design.

Jika jarak *(spacing)* faktor-faktor terlalu dekat, maka akan memperbesar kemungkinan bahwa *noise* akan membanjiri sinyal data. Jarak yang lebih agresif biasanya lebih baik.

Contoh: Eksperimen Resin *(The Resin Plan Experiment)*

- Sebuah faktorial 2^4 digunakan untuk meneliti efek empat faktor pada tingkat filtrasi dari sebuah resin
- Faktor tersebut adalah *A* = *temperature*, *B* = *pressure*, *C* = *mole ratio*, *D*= *stirring rate*
- Eksperimen dilakukan dalam sebuah *pilot plant*

Contoh: Eksperimen Resin (lanjutan)

Table 6-10 Pilot Plant Filtration Rate Experiment

Run Number	Factor A	B	C	D	Run Label	Filtration Rate (gal/h)
1	−	−	−	−	(1)	45
2	+	−	−	−	*a*	71
3	−	+	−	−	*b*	48
4	+	+	−	−	*ab*	65
5	−	−	+	−	*c*	68
6	+	−	+	−	*ac*	60
7	−	+	+	−	*bc*	80
8	+	+	+	−	*abc*	65
9	−	−	−	+	*d*	43
10	+	−	−	+	*ad*	100
11	−	+	−	+	*bd*	45
12	+	+	−	+	*abd*	104
13	−	−	+	+	*cd*	75
14	+	−	+	+	*acd*	86
15	−	+	+	+	*bcd*	70
16	+	+	+	+	*abcd*	96

Contoh: Eksperimen Resin (lanjutan)

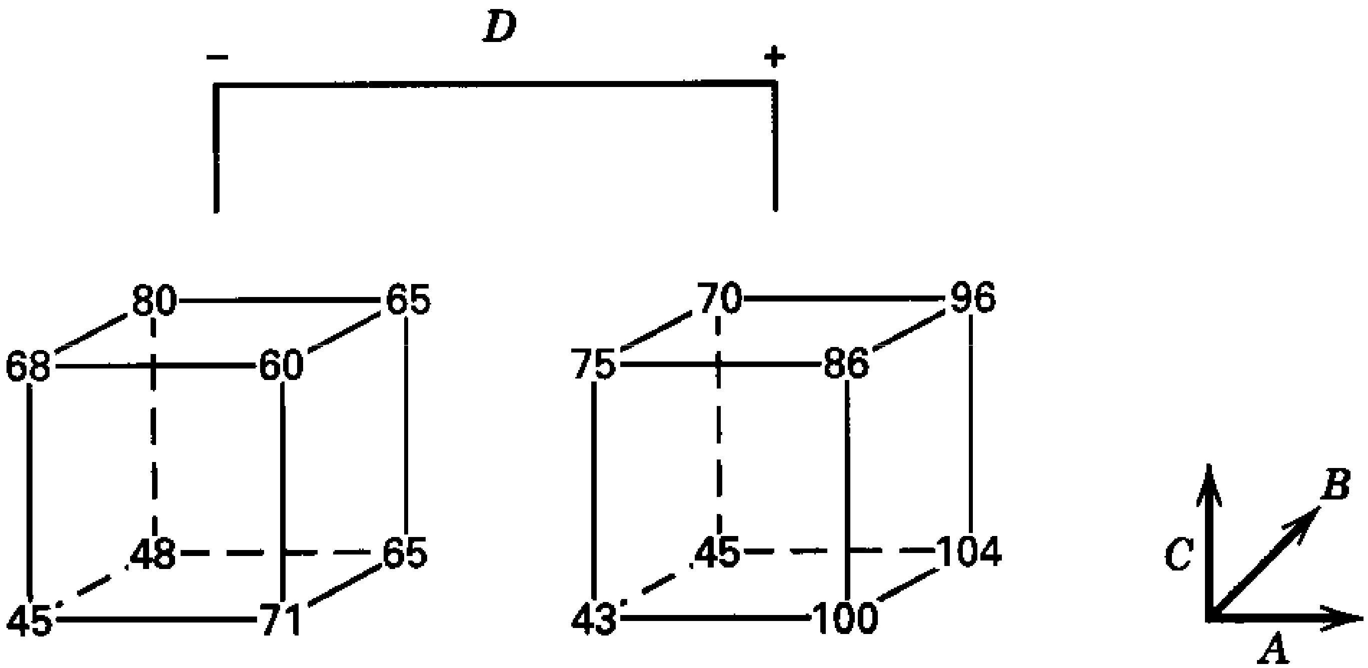


Figure 6-10 Data from the pilot plant filtration rate experiment for Example 6-2.

Estimasi efek faktor dan *Sums of Squares*

Table 6-12 Factor Effect Estimates and Sums of Squares for the 2^4 Factorial in Example 6-2

Model Term	Effect Estimate	Sum of Squares	Percent Contribution
A	21.625	1870.56	32.6397
B	3.125	39.0625	0.681608
C	9.875	390.062	6.80626
D	14.625	855.563	14.9288
AB	0.125	0.0625	0.00109057
AC	−18.125	1314.06	22.9293
AD	16.625	1105.56	19.2911
BC	2.375	22.5625	0.393696
BD	−0.375	0.5625	0.00981515
CD	−1.125	5.0625	0.0883363
ABC	1.875	14.0625	0.245379
ABD	4.125	68.0625	1.18763
ACD	−1.625	10.5625	0.184307
BCD	−2.625	27.5625	0.480942
ABCD	1.375	7.5625	0.131959

Plot Probabilitas Normal Efek

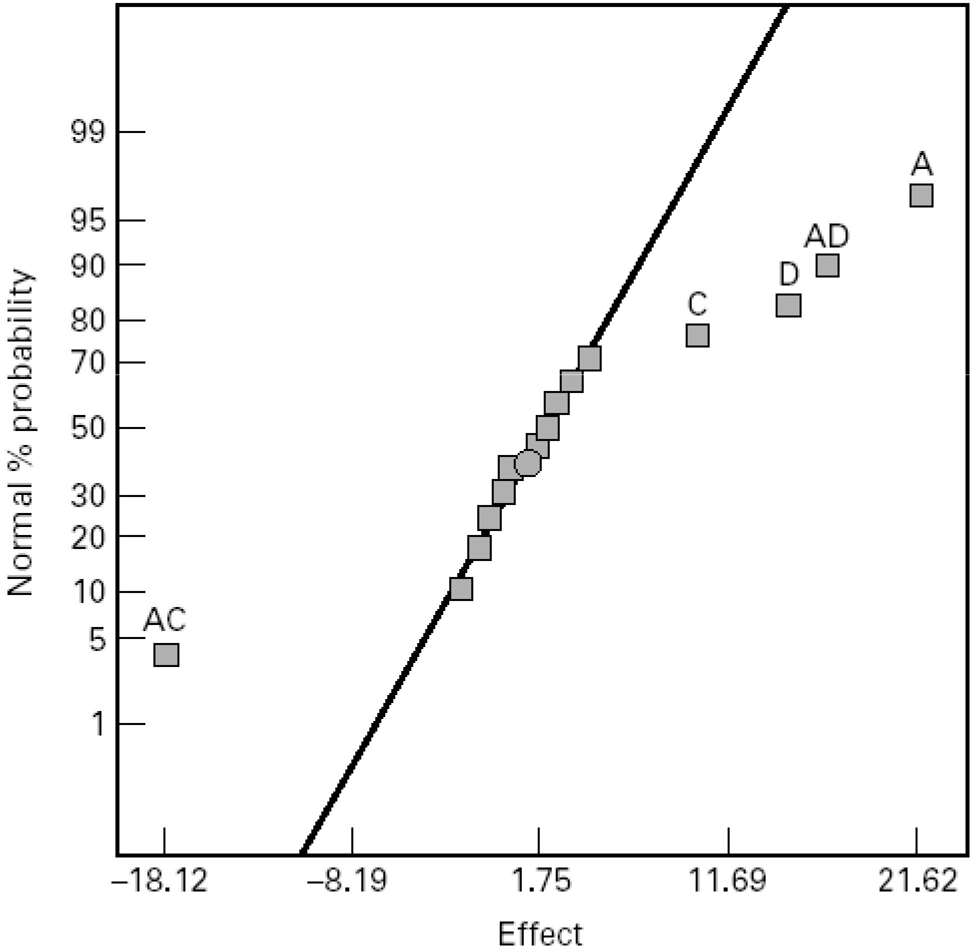


Figure 6-11 Normal probability plot of the effects for the 2^4 factorial in Example 6-2.

Ringkasan ANOVA dari Model

Table 6-13 Analysis of Variance for the Pilot Plant Filtration Rate Experiment in A, C, and *D*

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	*P*-Value
A	1870.56	1	1870.56	83.36	<0.0001
C	390.06	1	390.06	17.38	<0.0001
D	855.56	1	855.56	38.13	<0.0001
AC	1314.06	1	1314.06	58.56	<0.0001
AD	1105.56	1	1105.56	49.27	<0.0001
CD	5.06	1	5.06	<1	
ACD	10.56	1	10.56	<1	
Error	179.52	8	22.44		
Total	5730.94	15			

Interprestasi Model – Plot Permukaan Respon

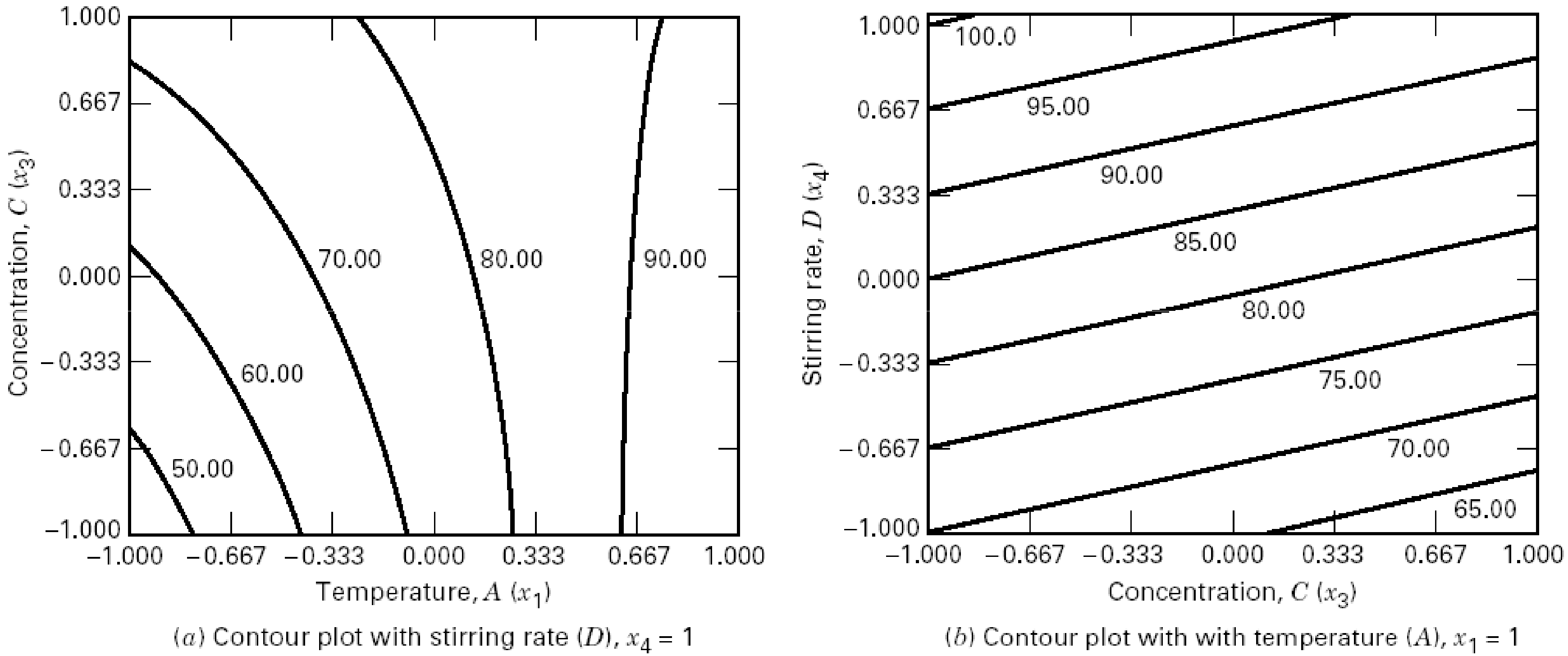


Figure 6-14 Contour plots of filtration rate, Example 6-2.

Pada konsentrasi tingkat rendah maupun tinggi, temperatur tinggi dan *stirring rate* tinggi menghasilkan tingkat filtrasi yang tinggi

3. b. Penambahan Titik Central *(Center Points)*

- Melakukan replikasi hanya pada **beberapa** percobaan dalam sebuah rancangan faktorial
- Percobaan pada titik pusat dapat menghasilkan estimasi error dan membedaan dua buah model:

First-order model (interaction) $y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \sum_{j>i}^{k} \beta_{ij} x_i x_j + \varepsilon$

Second-order model $y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \sum_{j>i}^{k} \beta_{ij} x_i x_j + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \varepsilon$

Penambahan *Center Points* pada Rancangan 2^k (lanjutan)

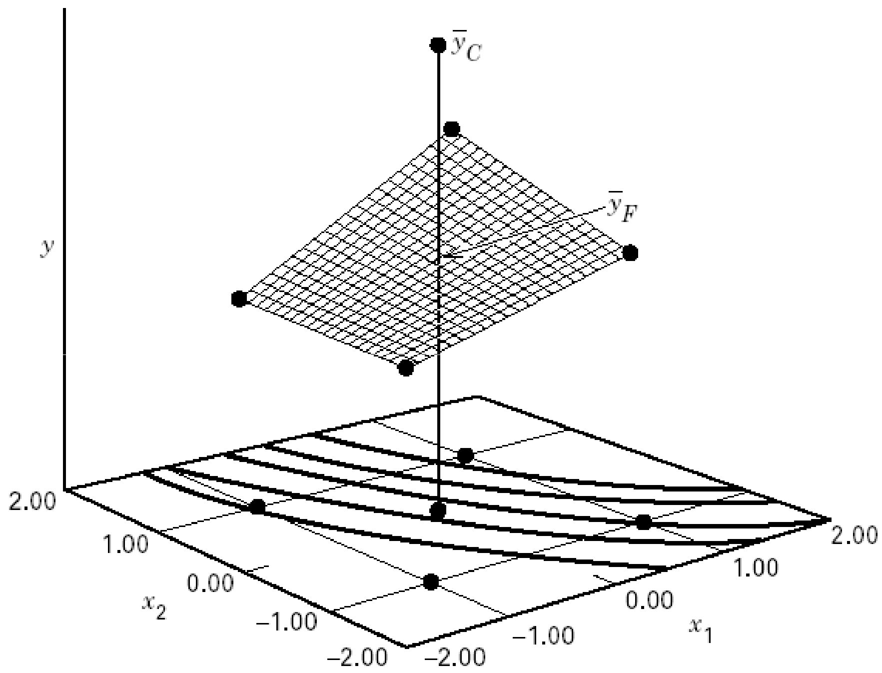


Figure 6-34 A 2^2 design with center points.

$$\bar{y}_F = \bar{y}_C \Rightarrow \text{ no "curvature"}$$

The hypotheses are:

$$H_0 : \sum_{i=1}^{k} \beta_{ii} = 0$$

$$H_1 : \sum_{i=1}^{k} \beta_{ii} \neq 0$$

$$SS_{\text{Pure Quad}} = \frac{n_F n_C (\bar{y}_F - \bar{y}_C)^2}{n_F + n_C}$$

This sum of squares has a single degree of freedom

Contoh

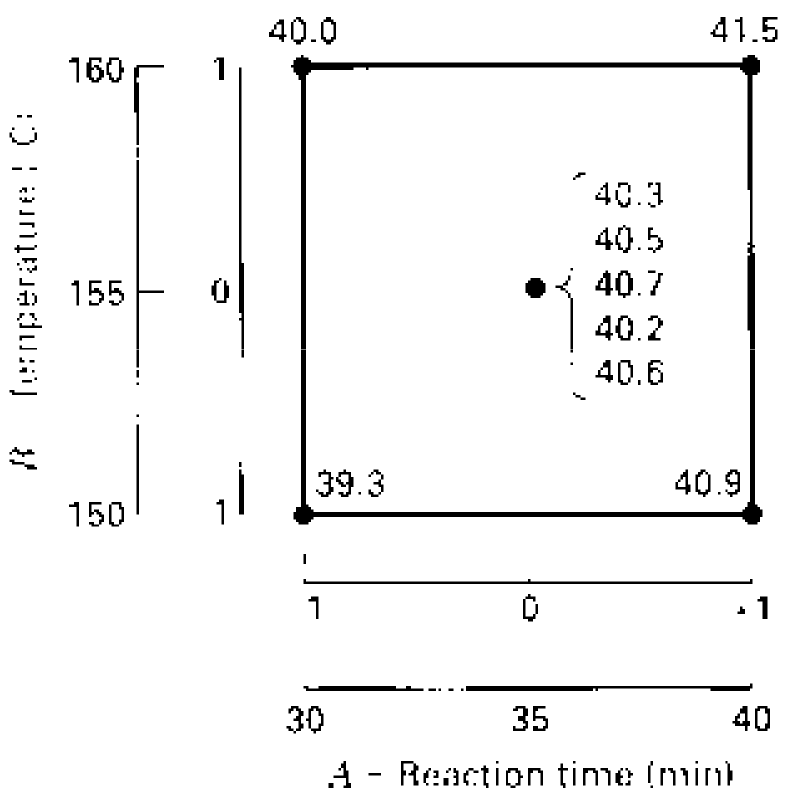


Figure 6-35 The 2^2 design with five center points for Example 6-6

$$n_C = 5$$

Biasanya dengan menggunakan 3 sampai 6 *center points*

Central Composite designs

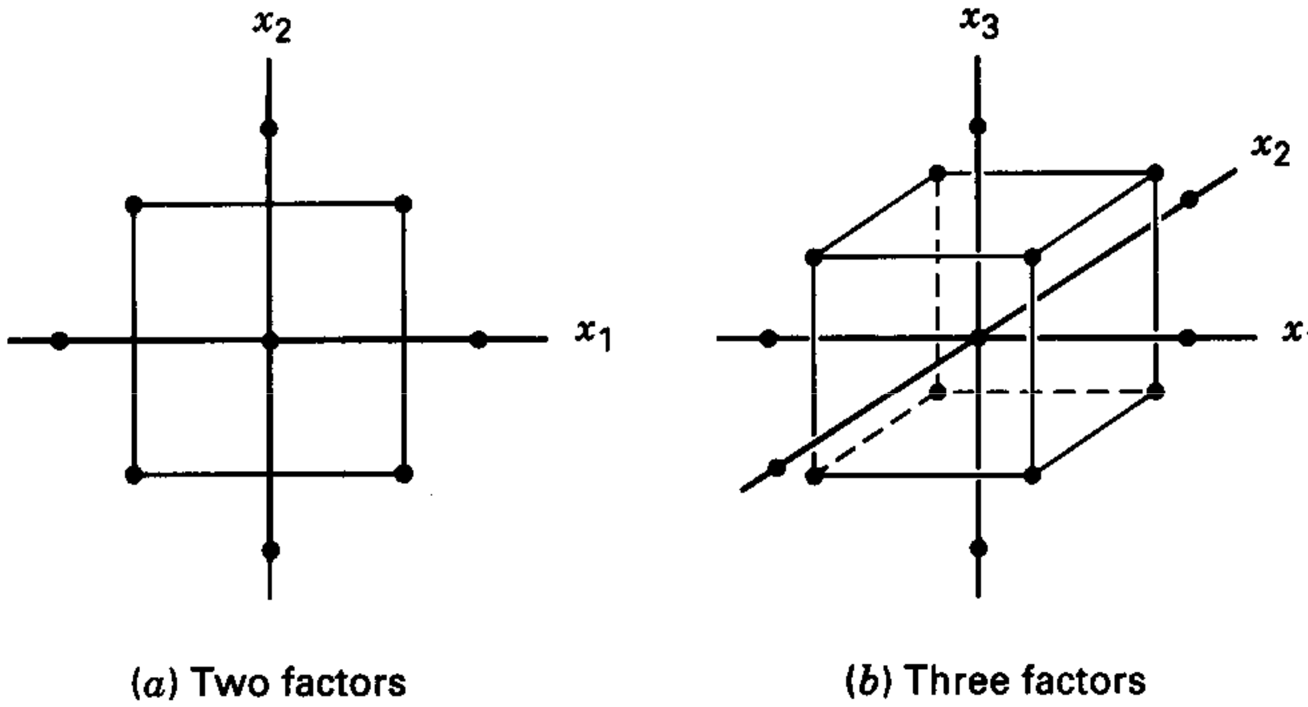


Figure 6-36 Central composite designs.

If curvature is significant, augment the design with axial runs to create a central composite design. The CCD is a very effective design for fitting a second-order response surface model

Penggunaan Praktis *Central Points*

- Gunakan **kondisi operasi saat ini** sebagai *center point*
- Terapkan pada kondisi **"abnormal"** percobaan
- Periksalah ***time trends***
- Gunakan *center point* dalam beberapa percobaan awal saat tidak ada atau sedikit informasi yang tersedia tentang besarnya error
- *Center points* dan faktor **kualitatif**?

Center Point dan Faktor Kualitatif

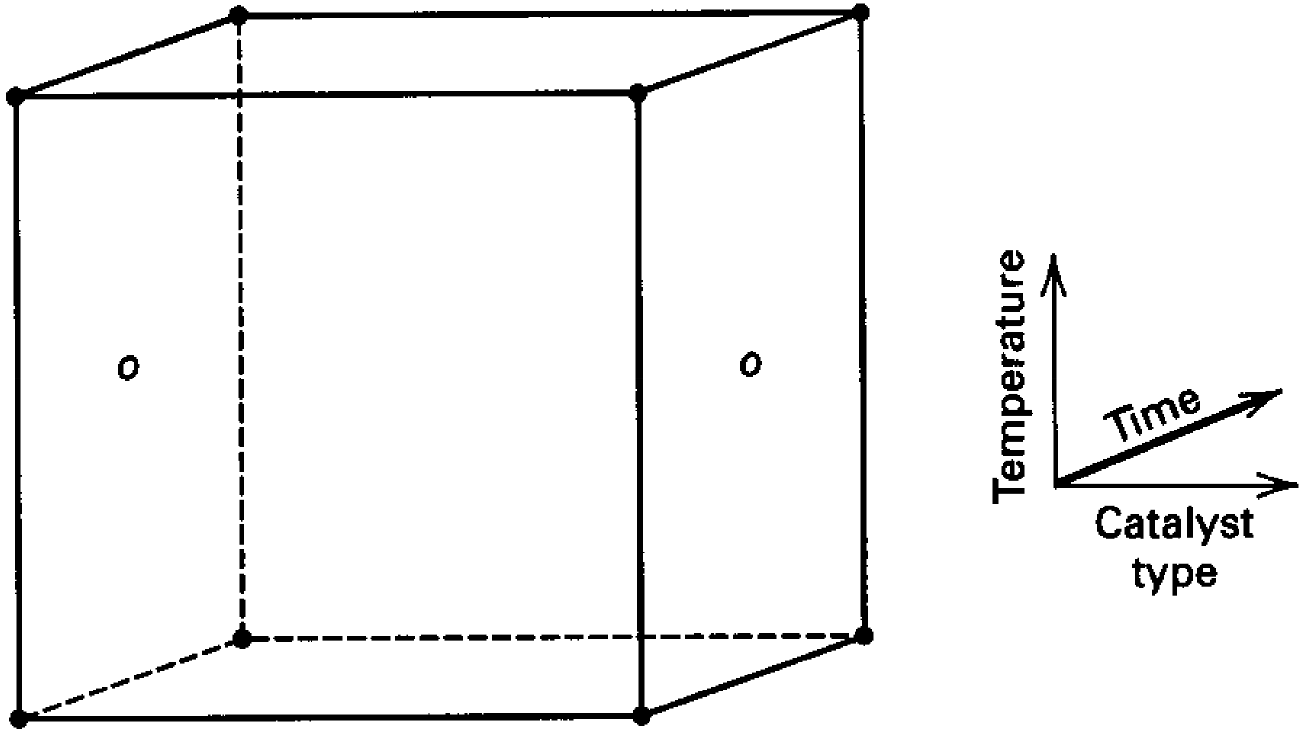


Figure 6-37 A 2^3 factorial design with one qualitative factor and center points.