Sumber: Harrell, C., B.K. Ghosh and R.O. Bowden, Jr., Simulation Using Promodel, 2nd ed., McGraw-Hill, Singapore, 2003.

BAB 9::
MEMBANDINGKAN SISTEM

MONICA A. KAPPIANTARI -2009

Pemodelan dan Simulasi Sistem

Bab 9:

Membandingkan Sistem

Bacaan:

- Harrell, Bab 10

Topik

- Uji Hipotesis
- Membandingkan dua rancangan alternatif sistem
- Membandingkan lebih dari dua rancangan alternatif sistem

1. Pendahuluan

- Dalam beberapa kasus, simulasi digunakan untuk membandingkan dua atau lebih rancangan alternatif dari sebuah sistem
- Tujuannya: untuk mengetahui suatu sistem relatif memiliki kinerja lebih baik dibanding sistem yang lain
- Metoda yang digunakan adalah **uji hipotesis**

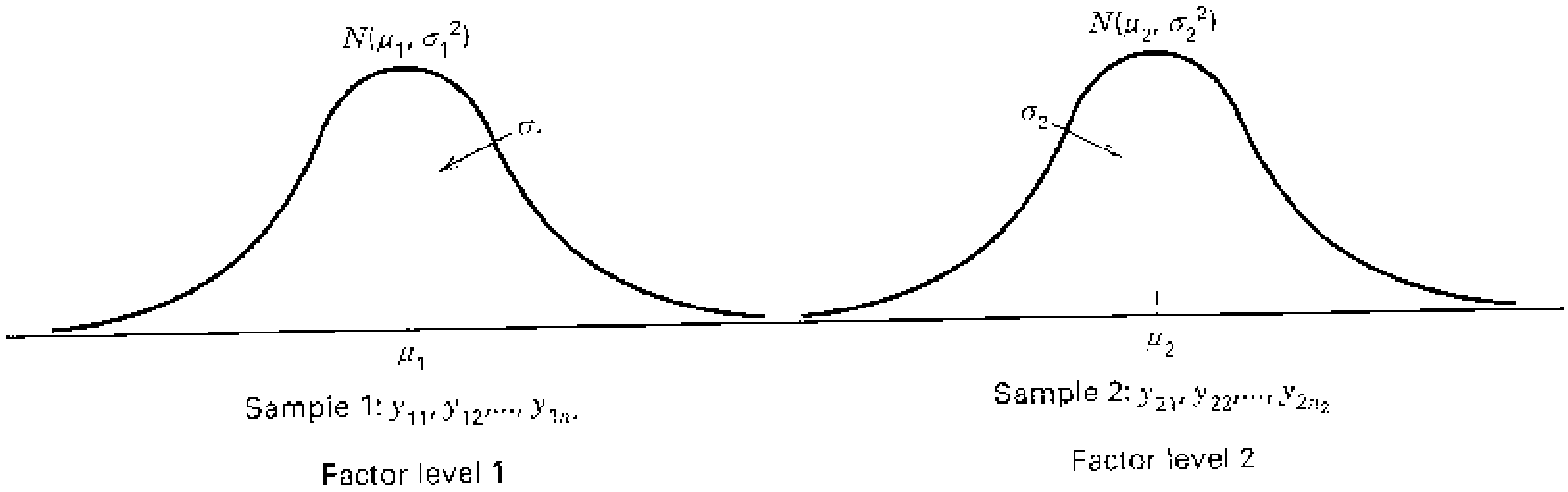
2. Uji Hipotesis

Uji Hipotesis

Sebuah hipotesis statistik

- Sebuah pernyataan atau dugaan terhadap satu atau lebih populasi
- Sebuah pernyataan tentang parameter-parameter dari sebuah distribusi probabilitas atau parameter dari sebuah model

Uji Hipotesis (lanjutan)



Uji Hipotesis (lanjutan)

Istilah-istilah penting

- Hipotesis nol
- Hipotesis alternatif
- Daerah kritis
- Daerah penolakan
- Tingkat signifikan *(Significance level (α))*
- Hipotesis alternatif dua-arah vs. satu-arah

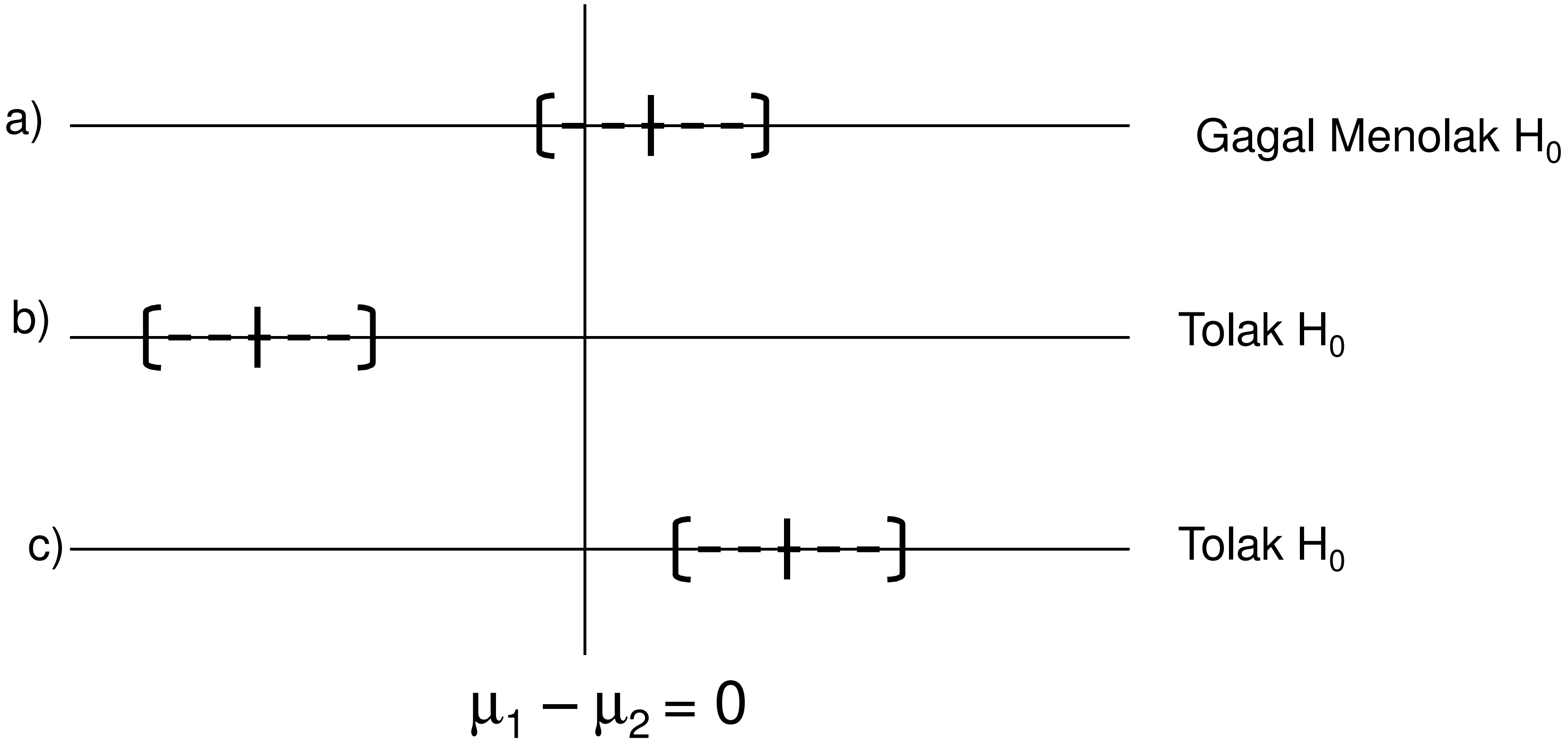
Uji Hipotesis (lanjutan)

- Penolakan hipotesis nol padahal hipotesis tersebut benar disebut kesalahan tipe I (α)
- Penerimaan hipotesis nol padahal hipotesis tersebut salah disebut kesalahan tipe II (β)

Table 8.1 Possible Situations in Testing a Statistical Hypothesis

	H_0 Is True	H_0 Is False
Accept H_0	Correct decision	Type II error
Reject H_0	Type I error	Correct decision

Tiga posisi tingkat kepercayaan (relatif terhadap nol)



[- - + - -] Confidence interval/tingkat kepercayaan $(x_1 - x_2) + hw$

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3. Membandingkan dua rancangan alternatif sistem

Metoda

- *Welch Confidence Interval*
- *Paired-t Confidence Interval*

Contoh

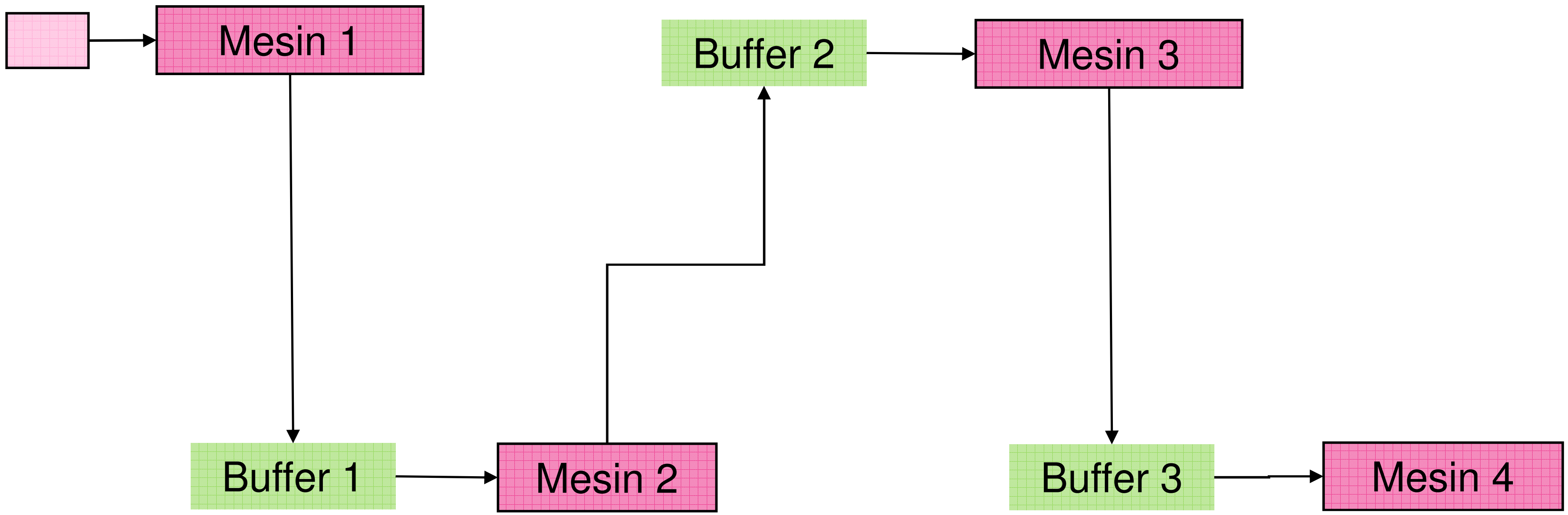
Sebuah sistem produksi memiliki 4 mesin dan 3 area penyimpanan penyangga *(buffer storage areas).* Komponen masuk ke sistem untuk diproses oleh keempat mesin secara serial.

Sebuah komponen selalu tersedia untuk diproses pada mesin pertama. Setelah diproses, komponen tersebut bergerak dari mesin ke penyangga untuk mesin berikutnya, menunggu untuk diproses.

Jika penyangga penuh, part tersebut tidak dapat bergerak ke mesin berikutnya dan tetap di mesin ybs, sampai tersedia tempat dalam penyangga. Selain itu, mesin juga tidak dapat memproses komponen lain (di-*block*). Komponen keluar dari sistem setelah diproses oleh mesin keempat.

Pertanyaan:

Bagaimana cara terbaik untuk mengalokasikan persediaan penyangga *(buffer storage)* diantara dua mesin untuk memaksimumkan kinerja sistem tersebut?


Mesin 1
Buffer 1
Mesin 2
Buffer 2
Mesin 3
Buffer 3
Mesin 4

Contoh (lanjutan)

- Staf Pengendalian Produksi telah mengidentifikasikan 2 strategi: strategi1 dan strategi2
- Simulasi dilakukan untuk 16 hari (24 jam per hari) untuk masing-masing strategi
- Simulasi direplikasi 10 kali untuk masing-masing strategi (ukuran sampel = 10)

Contoh (lanjutan)

TABLE 10.1 Comparison of Two Buffer Allocation Strategies

(A) Replication	(B) Strategy 1 Throughput x_1	(C) Strategy 2 Throughput x_2
1	54.48	56.01
2	57.36	54.08
3	54.81	52.14
4	56.20	53.49
5	54.83	55.49
6	57.69	55.00
7	58.33	54.88
8	57.19	54.47
9	56.84	54.93
10	55.29	55.84
Sample mean $\bar{x}_i$, for $i = 1, 2$	56.30	54.63
Sample standard deviation s_i, for $i = 1, 2$	1.37	1.17
Sample variance s_i^2, for $i = 1, 2$	1.89	1.36

Welch Confidence Interval

- Observasi diambil dari masing-masing populasi:
  - Terdistribusi normal
  - Independen dalam/antar populasi
- Jumlah sampel dari satu populasi **tidak** harus sama dengan populasi yang lain
- Dua populasi **tidak** harus memiliki variansi yang sama

Paired-t Confidence Interval

- Observasi diambil dari masing-masing populasi:
  - Terdistribusi normal
  - Independen dalam populasi, tapi **tidak** perlu independen antar populasi
- Jumlah sampel dari satu populasi dengan populasi lain harus sama
- Dua populasi **tidak** perlu memiliki variansi sama

Contoh

TABLE 10.2 Comparison of Two Buffer Allocation Strategies Based on the Paired Differences

(A) Replication (j)	(B) Strategy 1 Throughput x_{1j}	(C) Strategy 2 Throughput x_{2j}	(D) Throughput Difference (B − C) $x_{(1-2)j} = x_{1j} - x_{2j}$
1	54.48	56.01	−1.53
2	57.36	54.08	3.28
3	54.81	52.14	2.67
4	56.20	53.49	2.71
5	54.83	55.49	−0.66
6	57.69	55.00	2.69
7	58.33	54.88	3.45
8	57.19	54.47	2.72
9	56.84	54.93	1.91
10	55.29	55.84	−0.55
Sample mean $\bar{x}_{(1-2)}$			1.67
Sample standard deviation $s_{(1-2)}$			1.85
Sample variance $s^2_{(1-2)}$			3.42

4. Membandingkan lebih dari dua rancangan alternatif sistem

Metoda

- *The Bonferroni Approach*
- *Advanced Statistical Models:*
  - *ANOVA*
  - *Multiple Comparison Test (LSD)*
- *Factorial Design*

The Bonferroni Approach

- *Number of pairwise comparisons for K candidate designs is computed by K(K-1)/2*

The Bonferroni Approach

TABLE 10.3 Comparison of Three Buffer Allocation Strategies ($K = 3$) Based on Paired Differences

(A) Rep. (j)	(B) Strategy 1 Throughput x_{1j}	(C) Strategy 2 Throughput x_{2j}	(D) Strategy 3 Throughput x_{3j}	(E) Difference (B – C) Strategy 1 – Strategy 2 $x_{(1-2)j}$	(F) Difference (B – D) Strategy 1 – Strategy 3 $x_{(1-3)j}$	(G) Difference (C – Strategy 2 – Strate $x_{(2-3)j}$
1	54.48	56.01	57.22	−1.53	−2.74	−1.21
2	57.36	54.08	56.95	3.28	0.41	−2.87
3	54.81	52.14	58.30	2.67	−3.49	−6.16
4	56.20	53.49	56.11	2.71	0.09	−2.62
5	54.83	55.49	57.00	−0.66	−2.17	−1.51
6	57.69	55.00	57.83	2.69	−0.14	−2.83
7	58.33	54.88	56.99	3.45	1.34	−2.11
8	57.19	54.47	57.64	2.72	−0.45	−3.17
9	56.84	54.93	58.07	1.91	−1.23	−3.14
10	55.29	55.84	57.81	−0.55	−2.52	−1.97
$\bar{x}_{(i-i')}$, for all i and i' between 1 and 3, with $i < i'$				1.67	−1.09	−2.76
$s_{(i-i')}$, for all i and i' between 1 and 3, with $i < i'$				1.85	1.58	1.37

Advanced Statistical Model

TABLE 10.4 Experimental Results and Summary Statistics for a Balanced Experimental Design

Replication (j)	*Strategy 1 Throughput* (x_{1j})	*Strategy 2 Throughput* (x_{2j})	*Strategy 3 Throughput* (x_{3j})
1	54.48	56.01	57.22
2	57.36	54.08	56.95
3	54.81	52.14	58.30
4	56.20	53.49	56.11
5	54.83	55.49	57.00
6	57.69	55.00	57.83
7	58.33	54.88	56.99
8	57.19	54.47	57.64
9	56.84	54.93	58.07
10	55.29	55.84	57.81
Sum $x_i = \sum_{j=1}^{n} x_{ij} = \sum_{j=1}^{10} x_{ij}$, for $i = 1, 2, 3$	563.02	546.33	573.92
Sample mean $\bar{x}_i = \frac{\sum_{j=1}^{n} x_{ij}}{n} = \frac{\sum_{j=1}^{10} x_{ij}}{10}$, for $i = 1, 2, 3$	56.30	54.63	57.39

Advanced Statistical Model - ANOVA

TABLE 10.5 Analysis of Variance Table

Source of Variation	*Degrees of Freedom*	*Sum of Squares*	*Mean Square*	F
Total (corrected)	$N - 1 = 29$	SSTC = 71.73		
Treatment (strategies)	$K - 1 = 2$	SST = 38.62	MST = 19.31	15.70
Error	$N - K = 27$	SSE = 33.11	MSE = 1.23	

$$F_{tabel} = F_{(2,27,0.05)} = 3.35$$

Advanced Statistical Model - LSD

TABLE 10.6 LSD Analysis

	Strategy 2 $\bar{x}_2 = 54.63$	*Strategy 1* $\bar{x}_1 = 56.30$
Strategy 3 $\bar{x}_3 = 57.39$	$\|\bar{x}_2 - \bar{x}_3\| = 2.76$ Significant $(2.76 > 1.02)$	$\|\bar{x}_1 - \bar{x}_3\| = 1.09$ Significant $(1.09 > 1.02)$
Strategy 1 $\bar{x}_1 = 56.30$	$\|\bar{x}_1 - \bar{x}_2\| = 1.67$ Significant $(1.67 > 1.02)$	

Welch Confidence Interval

The Welch confidence interval for an α level of significance is

$$P[(\bar{x}_1 - \bar{x}_2) - hw \le \mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + hw] = 1 - \alpha$$

Half width = $$hw = t_{df,\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$df \approx \frac{[s_1^2/n_1 + s_2^2/n_2]^2}{[s_1^2/n_1]^2/(n_1 - 1) + [s_2^2/n_2]^2/(n_2 - 1)}$$

Welch Confidence Interval

See table 10.1: $\alpha = 0.05$

$$\mathrm{df} \approx \frac{[1.89/10 + 1.36/10]^2}{[1.89/10]^2/(10-1) + [1.36/10]^2/(10-1)} \approx 17.5$$

$$hw = t_{17.5,0.025}\sqrt{\frac{1.89}{10} + \frac{1.36}{10}} = 2.106\sqrt{0.325} = 1.20 \text{ parts per hour}$$

$$(\bar{x}_1 - \bar{x}_2) - hw \leq \mu_1 - \mu_2 \leq (\bar{x}_1 - \bar{x}_2) + hw$$

$$(56.30 - 54.63) - 1.20 \leq \mu_1 - \mu_2 \leq (56.30 - 54.63) + 1.20$$

$$0.47 \leq \mu_1 - \mu_2 \leq 2.87$$

→ Reject H_0, choose strategy1

Paired-t Confidence Intervali

$$\text{Sample mean} = \bar{x}_{(1-2)} = \frac{\sum_{j=1}^{n} x_{(1-2)j}}{n}$$

$$\text{Sample standard deviation} = s_{(1-2)} = \sqrt{\frac{\sum_{j=1}^{n} \left[x_{(1-2)j} - \bar{x}_{(1-2)}\right]^2}{n-1}}$$

where $\bar{x}_{(1-2)}$ estimates $\mu_{(1-2)}$ and $s_{(1-2)}$ estimates $\sigma_{(1-2)}$.

$$hw = \frac{(t_{n-1,\alpha/2})s_{(1-2)}}{\sqrt{n}}$$

$$P\left(\bar{x}_{(1-2)} - hw \le \mu_{(1-2)} \le \bar{x}_{(1-2)} + hw\right) = 1 - \alpha$$

Paired-t Confidence Interval

See table 10.2: $\alpha = 0.05$

$$\bar{x}_{(1-2)} = \frac{\sum_{j=1}^{10} x_{(1-2)j}}{10} = 1.67 \text{ parts per hour}$$

$$s_{(1-2)} = \sqrt{\frac{\sum_{j=1}^{10} \left[x_{(1-2)j} - 1.67\right]^2}{10-1}} = 1.85 \text{ parts per hour}$$

$$hw = \frac{(t_{9,0.025})1.85}{\sqrt{10}} = \frac{(2.262)1.85}{\sqrt{10}} = 1.32 \text{ parts per hour}$$

Paired-t Confidence Interval

$$\bar{x}_{(1-2)} - hw \le \mu_{(1-2)} \le \bar{x}_{(1-2)} + hw$$

$$1.67 - 1.32 \le \mu_{(1-2)} \le 1.67 + 1.32$$

$$0.35 \le \mu_{(1-2)} \le 2.99$$

→ Tolak H_0, pilih strategi1

Bonferroni Approach

H_0: $\mu_1 = \mu_2 = \mu_3 = \mu$

H_1: $\mu_1 \neq \mu_2$ or $\mu_1 \neq \mu_3$ or $\mu_2 \neq \mu_3$

$$\alpha_i = \frac{\alpha}{K(K-1)/2} \qquad \text{for } i = 1, 2, 3, \ldots, K(K-1)/2$$

$$\alpha_i = \frac{\alpha}{3} = \frac{0.06}{3} = 0.02 \qquad \text{for } i = 1, 2, 3$$

Bonferroni Approach

See table 10.3: α = 0.06

Comparing $\mu_{(1-2)}$: $\alpha_1 = 0.02$

$$t_{n-1,\alpha_1/2} = t_{9,0.01} = 2.821 \text{ from Appendix B}$$

$$hw = \frac{(t_{9,0.01})s_{(1-2)}}{\sqrt{n}} = \frac{(2.821)1.85}{\sqrt{10}}$$

$$hw = 1.65 \text{ parts per hour}$$

Bonferroni Approach

$$\bar{x}_{(1-2)} - hw \leq \mu_{(1-2)} \leq \bar{x}_{(1-2)} + hw$$

$$1.67 - 1.65 \leq \mu_{(1-2)} \leq 1.67 + 1.65$$

$$0.02 \leq \mu_{(1-2)} \leq 3.32$$

Comparing $\mu_{(1-3)}$: $\alpha_2 = 0.02$

$$t_{n-1,\alpha_2/2} = t_{9,0.01} = 2.821 \text{ from Appendix B}$$

$$hw = \frac{(t_{9,0.01})s_{(1-3)}}{\sqrt{n}} = \frac{(2.821)1.58}{\sqrt{10}}$$

$$hw = 1.41 \text{ parts per hour}$$

Bonferroni Approach

$$\bar{x}_{(1-3)} - hw \leq \mu_{(1-3)} \leq \bar{x}_{(1-3)} + hw$$

$$-1.09 - 1.41 \leq \mu_{(1-3)} \leq -1.09 + 1.41$$

$$-2.50 \leq \mu_{(1-3)} \leq 0.32$$

$$-3.98 \leq \mu_{(2-3)} \leq -1.54$$

Strategy2 tidak terlalu baik mengingat rata-rata *throughput*-nya